Effect of energy injection on jet-waves-random interactions across scales: theory

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Outline

1. Context and the question

Context of blocking

Problem: unravel the mechanisms of genesis, persistence, and dissipation of energy→ focus on extreme events

2. Methodology

First principles + Triple decomposition framework

First principles? Navier Stokes, advection-diffusion of temperature…

Which form? -constant physical properties of the fluid (constant density, viscosity, D)

-variable density

-variable viscosity

-T/NT interfaces

-variable molecular diffusivity- **differential diffusion**

-with phase changes…

3. Results: 2003 summer, next presentation

1. Context. The general context of climate

Climate and Its Sensitivity

Let's say CO_2 doubles:

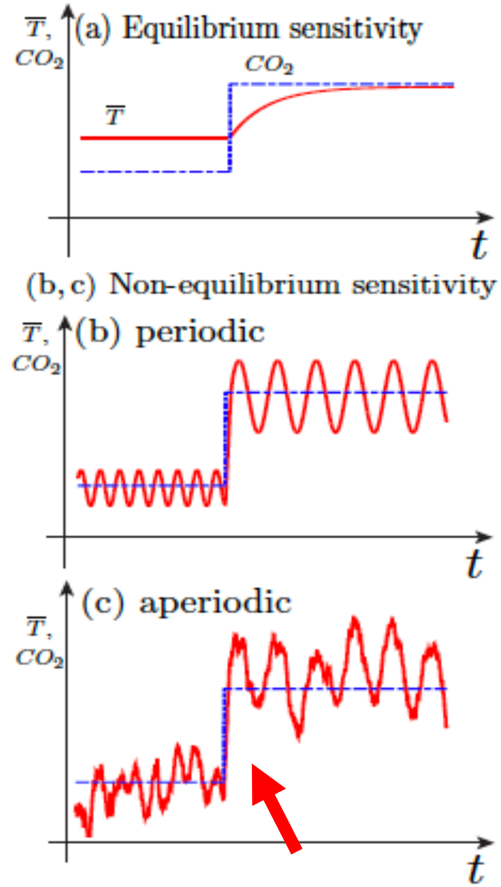
How will "climate" change?

1. Climate is in stable equilibrium (fixed point); if so, mean temperature will just shift gradually to its new equilibrium value.

2. Climate is purely periodic; if so, mean temperature will (maybe) shift gradually to its new equilibrium value. But how will the period, amplitude and phase of the limit cycle change?

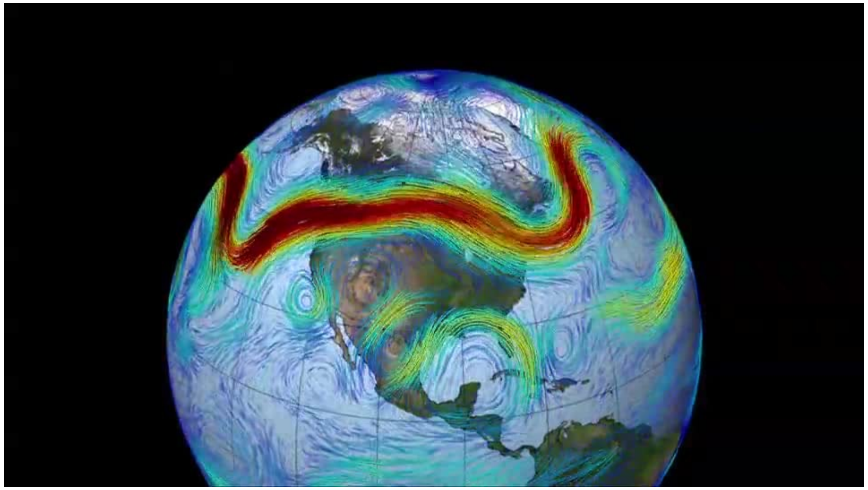
3. And how about some "real stuff" now: chaotic + random?

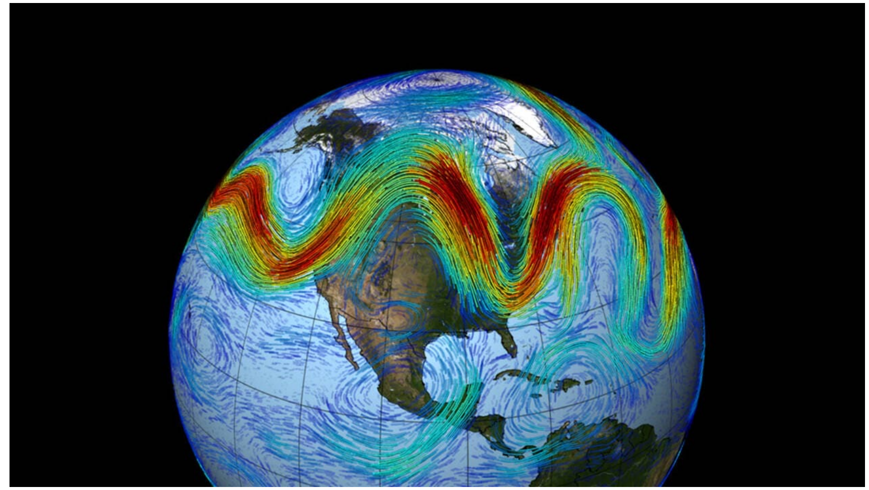
Ghil (in *Encycl. Global Environmental Change*, 2002)



Jump of fluctuations and statistics:
Need for d/dt (M_n)
Here: n=2, focus on production

1. Context. Blocking



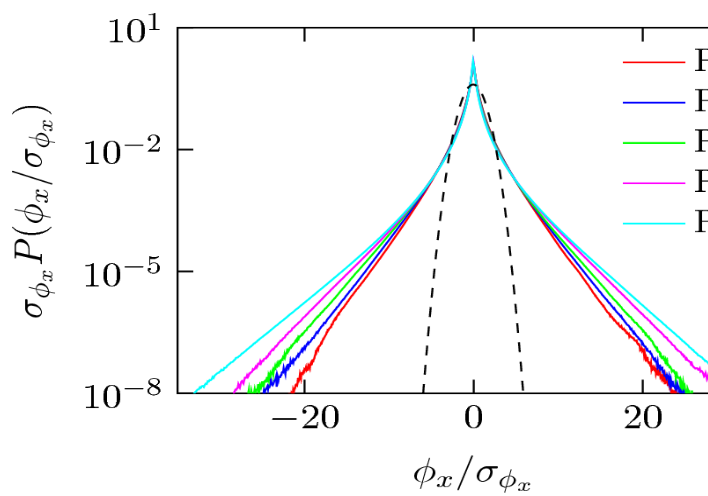


Blocking →Jet stream (periodic motion)→

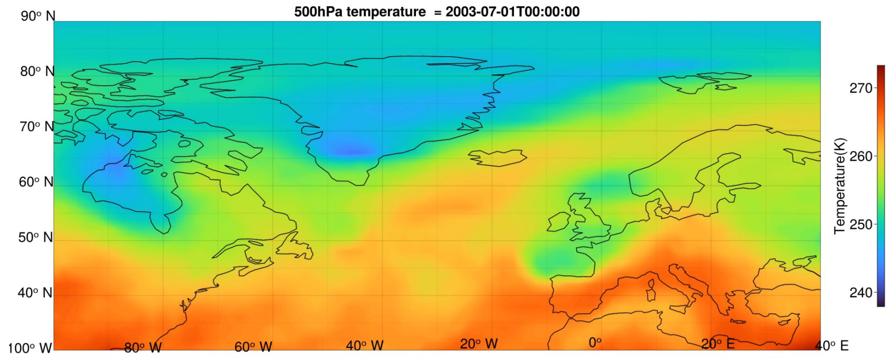
Local variability of the temperature gradient

Energy source/Production / dissipation/diffusion/transport terms

= transport equations for temperature fluctuation



1. Context. Blocking during the 2003 summer heat wave



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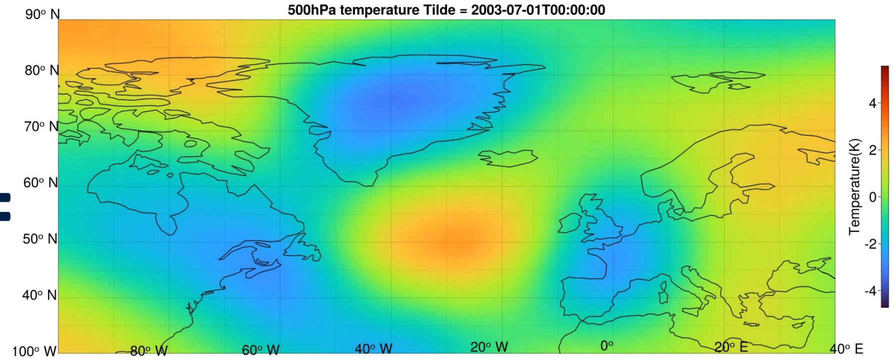

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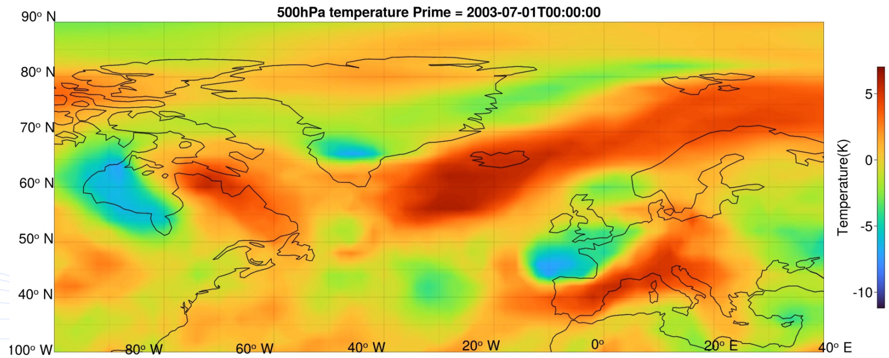
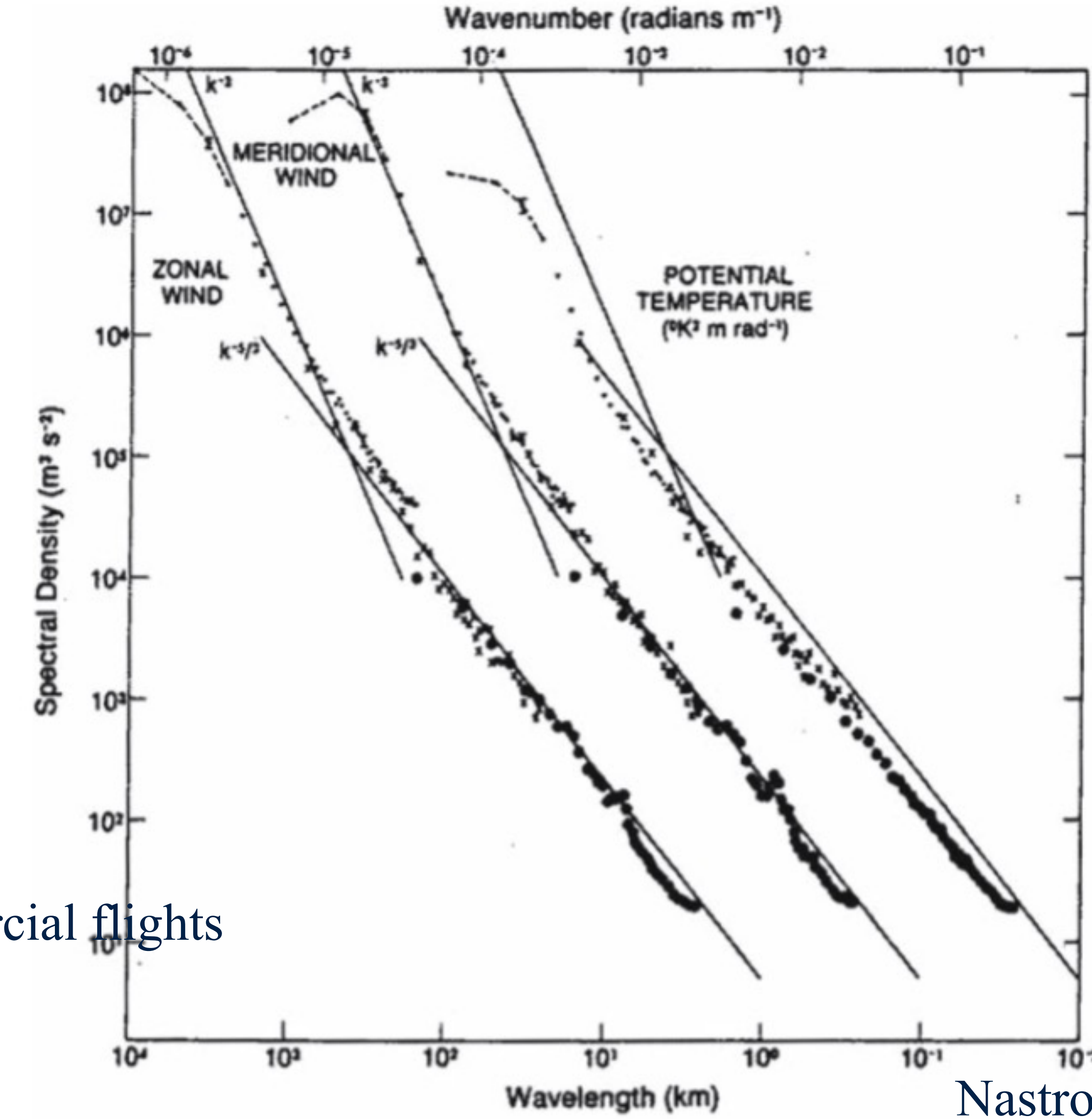


Mean + CM/Waves + Eddies = Turbulence at all spatiotemporal scales

1. Context. Scales: The MacroTurbulence



- Obs from commercial flights
- 9-12km altitude

Nastrom and Gage, 1985

All scales are present: different scalings, reflecting different physical mechanisms

1. Context. Scales: The MacroTurbulence

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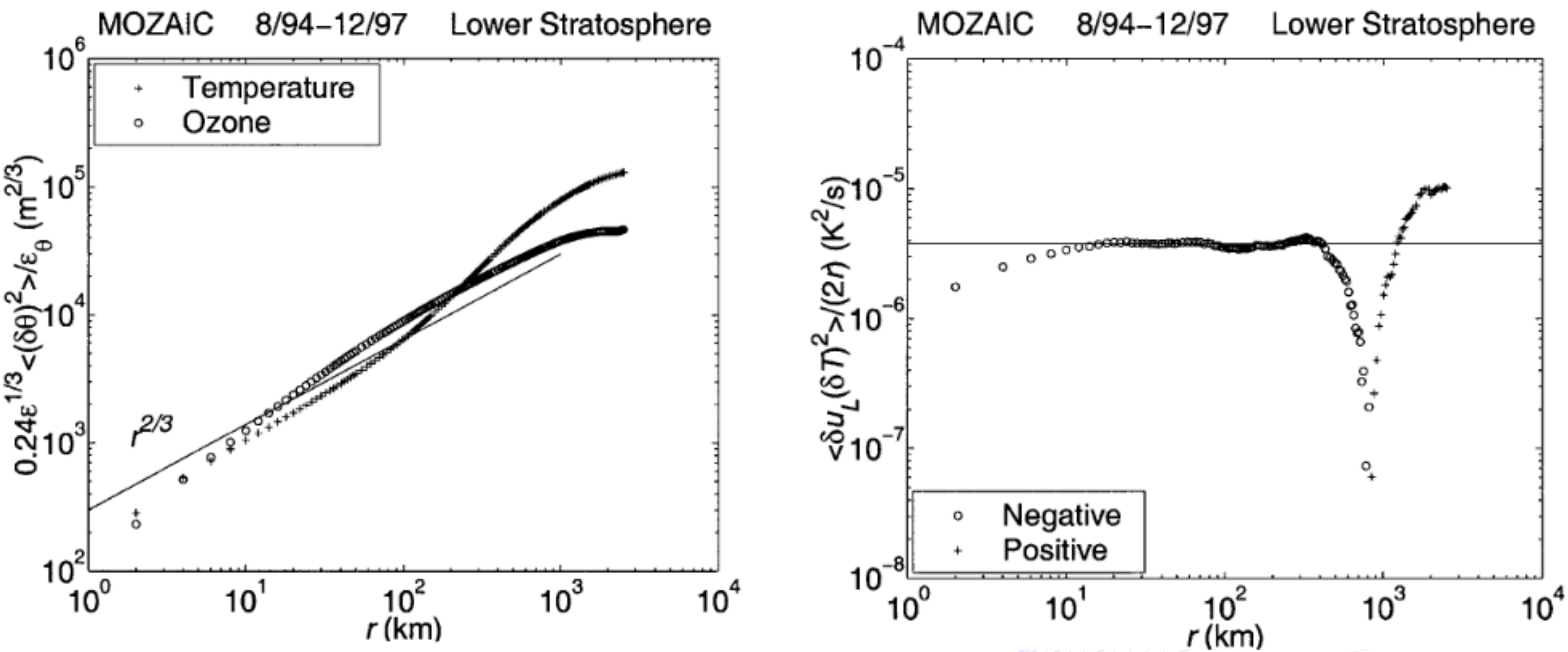


- Obs from commercial flights
- 9-12km altitude
- Temperature derived from velocity via Taylor hypothesis

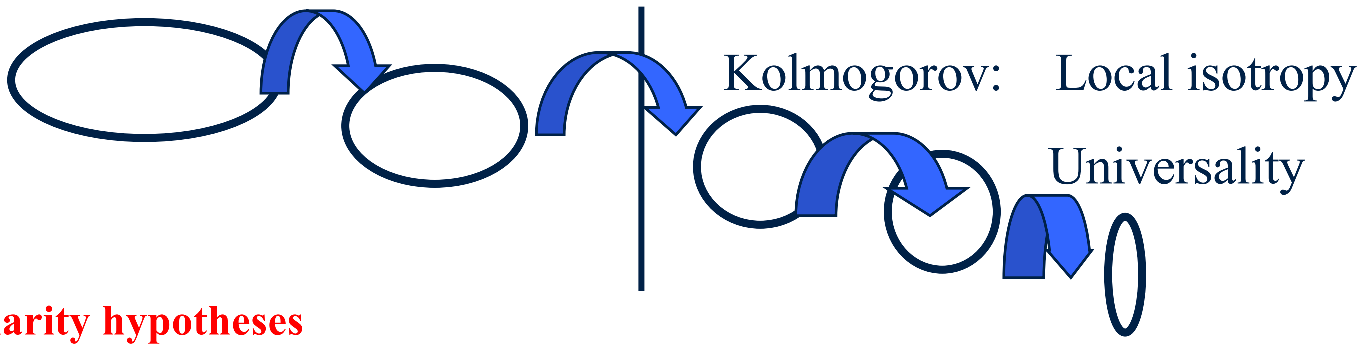
(Linborg and Cho, 2000)

2. Methodology to obtain Scale-by-Scale transport equations

Historical context and motivation

FLOW

Re



K41: Similarity hypotheses

HIGH Reynolds numbers

1st: $\overline{(\Delta u^*)^n} = f_{un}(r^*)$ $r^* = r/\eta$ $\eta \equiv \left(\frac{\nu^3}{\bar{\epsilon}}\right)^{\frac{1}{4}}$

2nd : for $\eta \ll r \ll L$ (L is the integral length scale),

$$\overline{(\Delta u^*)^n} = C_{un} r^{*n/3}$$

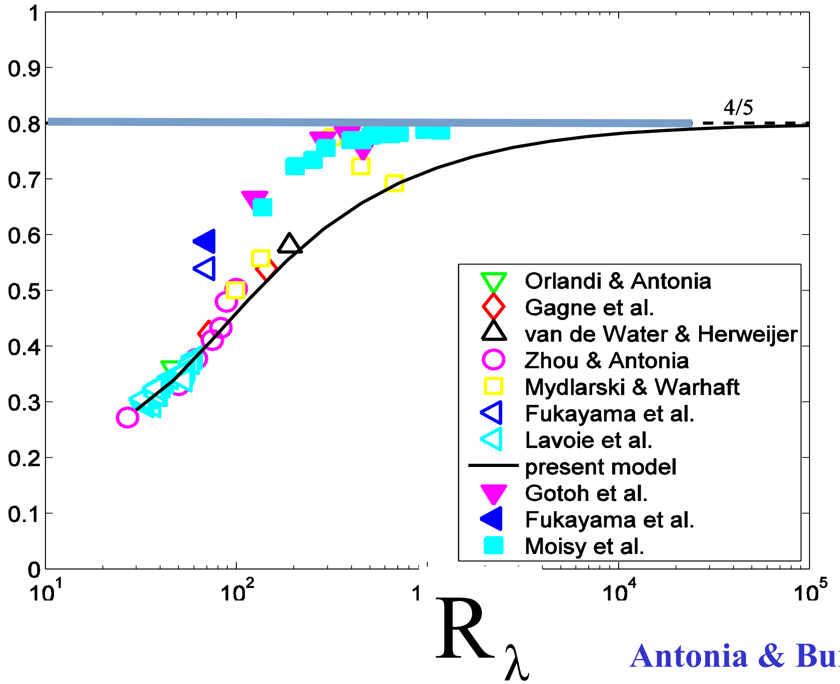
C_{un} = "universal" constants.

2. Methodology to obtain Scale-by-Scale transport equations

Historical context and motivation

Kolmogorov (1941) equation

$$-\frac{\overline{(\Delta u)^3)}}{\bar{\epsilon}\, r} + 6\nu \frac{\frac{d}{dr}\overline{(\Delta u)^2)}}{\bar{\epsilon}\, r} = \frac{4}{5}$$



HIGH Reynolds numbers

$$-\frac{\overline{(\Delta u)^3)}}{\bar{\epsilon}\, r} = \frac{4}{5}$$

Error: Mix-up of infinite Reynolds number phenomenology, with mathematics.

Non-universality for moderate Reynolds numbers

Antonia & Burattini, 2006

2. Methodology to obtain Scale-by-Scale transport equations

Finite Reynolds number effect

Similar questions hold for scalars and turbulent kinetic energy

$$\frac{4}{5} = 6\nu \frac{\frac{d}{dr}\overline{(\Delta u)^2)}}{\bar{\epsilon}\, r} - \frac{\overline{(\Delta u)^3)}}{\bar{\epsilon}\, r} + I_f$$

Kolmogorov, 1941

$$\frac{4}{3} = 2k \frac{\frac{d}{dr}\overline{(\Delta \theta)^2)}}{\bar{\chi}\ r} - \frac{\overline{\Delta u(\Delta \theta)^2)}}{\bar{\chi}\, r} + I_f$$

Yaglom, 1949
Danaila et al. 1999

$$\frac{4}{3} = 2\nu \frac{\frac{d}{dr}\overline{(\Delta q)^2)}}{\bar{\varepsilon}\ r} - \frac{\overline{\Delta u(\Delta q)^2)}}{\bar{\varepsilon}\, r} + I_f$$

R.A. Antonia et al. 1997
Danaila et al., 2004
Burattini et al., 2005

Real- Finite Reynolds numbers- flows: Slightly heated grid turbulence, grid turbulence with a Mean scalar gradient ..

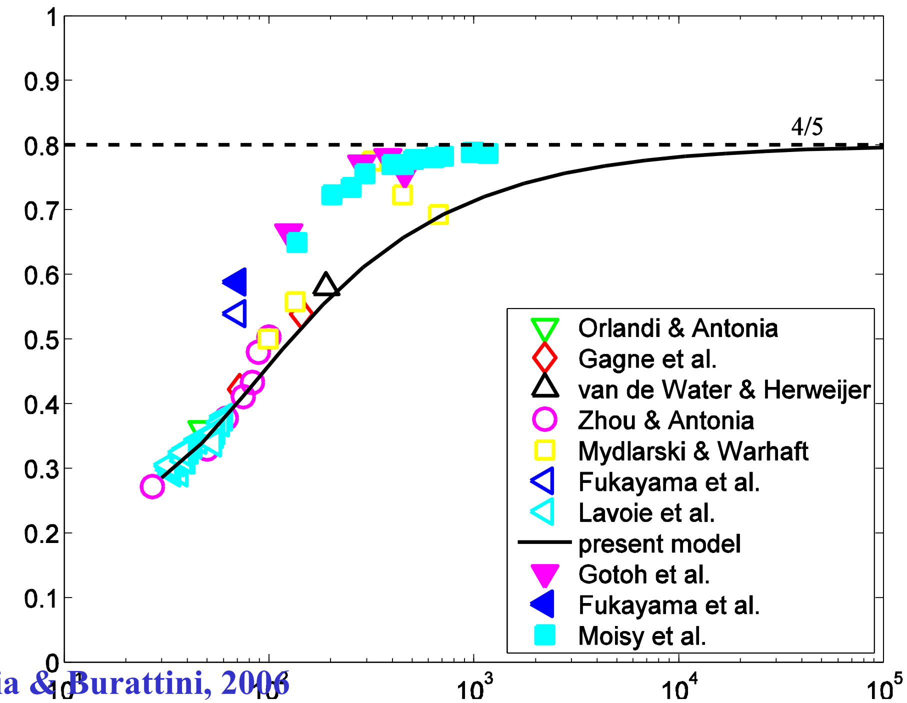
Same conclusion : Energy transferred at a scale r→ … large-scale effects

2. Methodology to obtain Scale-by-Scale transport equations

Finite Reynolds number effect

$$\frac{4}{5} = 6\nu \frac{\frac{d}{dr}\overline{(\Delta u)^2)}}{\overline{\epsilon}\, r} - \frac{\overline{(\Delta u)^3)}}{\overline{\epsilon}\, r} + I_f$$

Kolmogorov, 1941 → Saffman 1968, Danaila et al. 1999, Lindborg 1999



Antonia & Burattini, 2006

First conclusion:

Part of the K41 theory was rederived so it can be correctly applied to real, finite Reynolds numbers flows.

UNCLOSED Equation!!!

Other, complex flows →

2. Methodology to obtain Scale-by-Scale transport equations: high-order moments

$$\frac{\partial}{\partial t}\langle(\Delta\phi)^{2n}\rangle(\mathbf{r}) + \underbrace{\frac{\partial}{\partial r_i}\langle(\Delta u_i)(\Delta\phi)^{2n}\rangle}_{\text{transport term}}(\mathbf{r}) + \underbrace{2n\Gamma\langle(\Delta u_2)(\Delta\phi)^{2n-1}\rangle}_{\text{production term}}(\mathbf{r}) = J_{2n}(\mathbf{r})$$

$$J_{2n}(r) = nD\langle(\Delta\phi)^{n-1}\left[\frac{\partial^2(\Delta\phi)}{\partial x_i'^2} + \frac{\partial^2(\Delta\phi)}{\partial x_i^2}\right]\rangle$$

$$J_{2n}(r) = \underbrace{2D\frac{\partial^2}{\partial r_i^2}\langle(\Delta\phi)^{2n}\rangle}_{\text{diffusive transport}} - \underbrace{n(2n-1)\langle(\Delta\phi)^{2n-2}\left[\chi(\mathbf{x}+\mathbf{r}) + \chi(\mathbf{x})\right]\rangle}_{\text{dissipative source term}},$$

2nd order → $2\langle\chi\rangle$ $\neq f(r)$

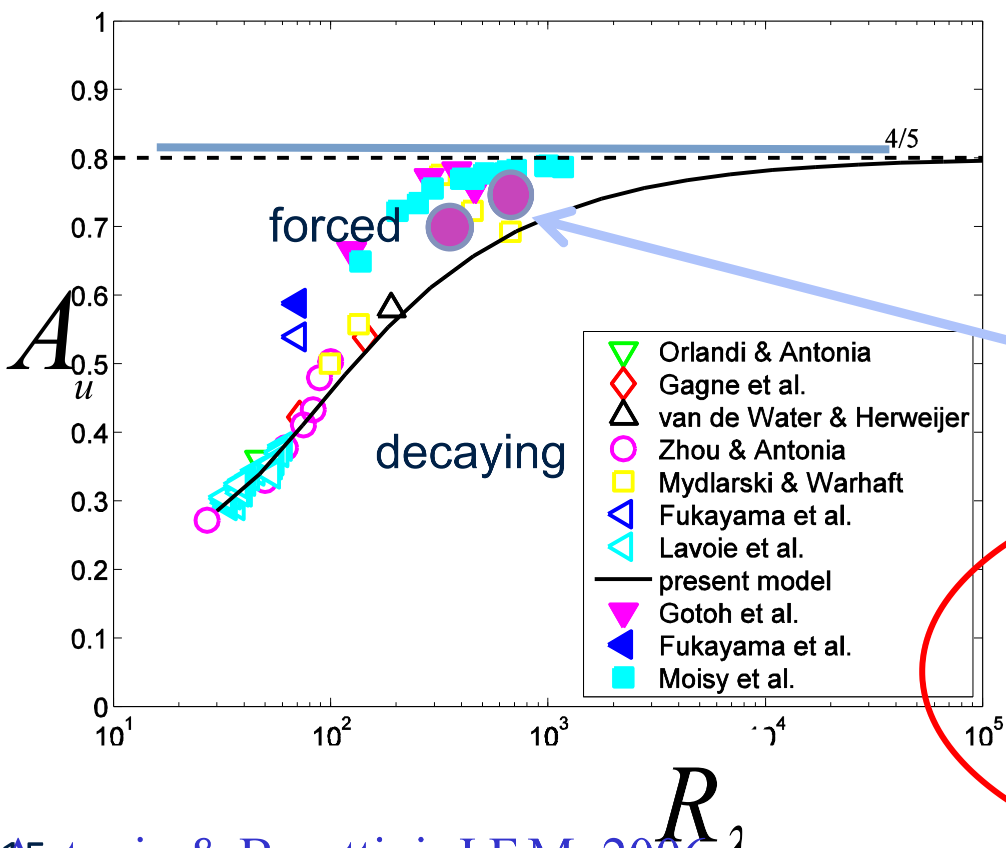
4th order → $12\langle(\Delta\phi)^2\chi\rangle$ $= f(r)$

CM???

2. Methodology to obtain Scale-by-Scale transport equations: flows with Coherent Motions

$$\frac{4}{5} = 6\nu \frac{\frac{d}{dr}\overline{(\Delta u)^2)}}{\bar{\epsilon}\, r} - \frac{\overline{(\Delta u)^3)}}{\bar{\epsilon}\, r} + I_f$$

➔**Non-universality for moderate Reynolds numbers**



→Forced, anisotropic

→Decaying flows: grid turbulence, jets ..

→Decaying, but populated by CM

Which is the role played by the mean shear and waves/CM in energy itself and energy transfer?

Antonia & Burattini, J.F.M. 2006

I. Background and major question

-interaction between coherent motion (CM)
turbulent/random motion (RM)
during energy transfer

Historically:

-identifications of CM [Hussain 1983, Reynolds & Hussain 1972, …].

-dynamics of CM, their representativity for turbulence ..

-From an analytical viewpoint, Reynolds et Hussain [*J. Fluid Mech.* 1972] derived the 1-point kinetic energy budget, including the coherent motion.

2. Methodology to obtain Scale-by-Scale transport equations: flows with Coherent Motions

New approach. Phase-averages

Triple decomposition [3]: $\beta = \overline{\beta} + \widetilde{\beta} + \beta'$

Phase-average: $\langle \beta \rangle = \overline{\beta} + \widetilde{\beta}$

Phase-averaged Strain: $\langle S \rangle = \overline{S} + \widetilde{S} = \frac{1}{2}\left(\frac{\partial \langle U \rangle}{\partial y} + \frac{\partial \langle V \rangle}{\partial x}\right)$

[3] Reynolds and Hussain 1972

2. Methodology to obtain Scale-by-Scale transport equations: flows with Coherent Motions

Transverse Velocity

$$v$$

Band pass filter

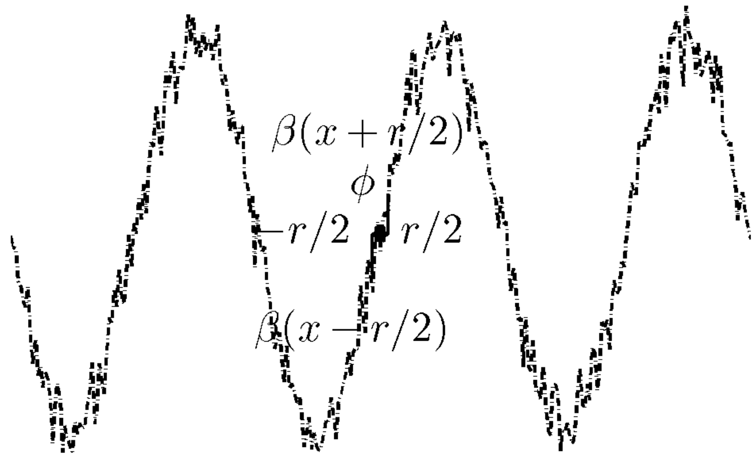
$$v_f$$

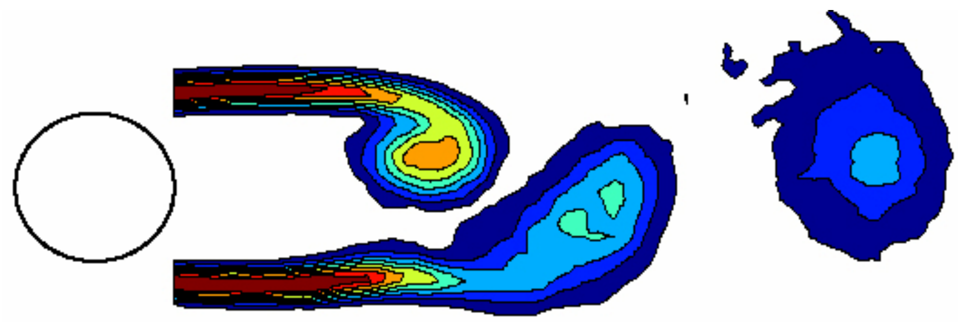
Hilbert Transform

$$h$$

Phase

$$\varphi = \tan^{-1}\left(\frac{h}{v_f}\right)$$





2. Methodology to obtain Scale-by-Scale transport equations: flows with Coherent Motions

$$\Delta\alpha(\vec{r}) = \alpha(\vec{x}+\vec{r}) - \alpha(\vec{x})$$

$$\langle \Delta q'^2 \rangle (\vec{r},\phi) = \langle (u_i'(\vec{x}+\vec{r}) - u_i'(\vec{x}))^2 \rangle$$

$$\Delta \tilde{q}^2 (\vec{r},\phi) = (\tilde{u}_i(\vec{x}+\vec{r}) - \tilde{u}_i(\vec{x}))^2$$

Objectives:

1) First, we develop transport equations –
   using both general and isotropic formulations for these 2 quan
2) Second, we analyze these statistics.

2. Methodology. Transport equations

Analytical considerations [Antonia et al JFM 97, Danaila et al JFM 99]

3 . Each equation is considered at point $\vec{x}$ and $\vec{x}+\vec{r}$

then subtracted

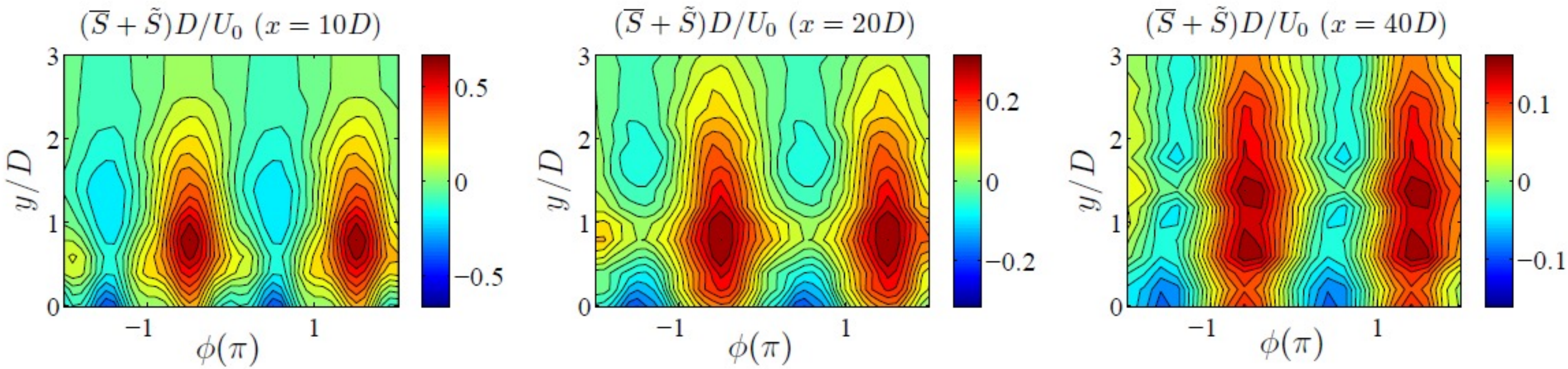
4 . The equation of CM is multiplied by $2\Delta\tilde{u}_i$
And that of random motion by $2\Delta u'_i$

5 . A phase averaging ………
Assess the temporal dynamics associated with CM, one step before time averaging

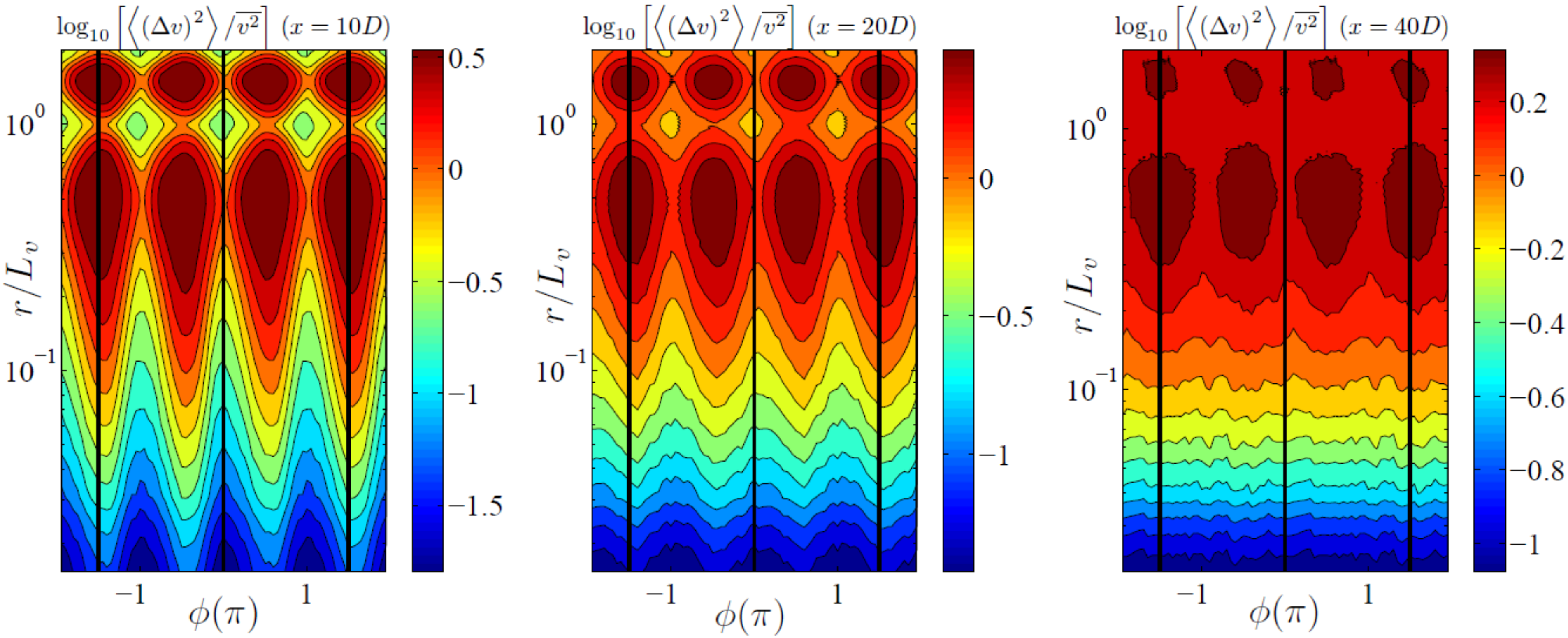
Thiesset, Danaila and Antonia, J. Fluid Mech. 2013, 2014

2. Assessment of Scale-by-Scale transport equations: Flows with CM

Coherent strain $\tilde{S} = \left(\frac{\partial \tilde{u}}{\partial y} + \frac{\partial \tilde{v}}{\partial x}\right)$ normalized by D/U_0 in the (ϕ, y) plane



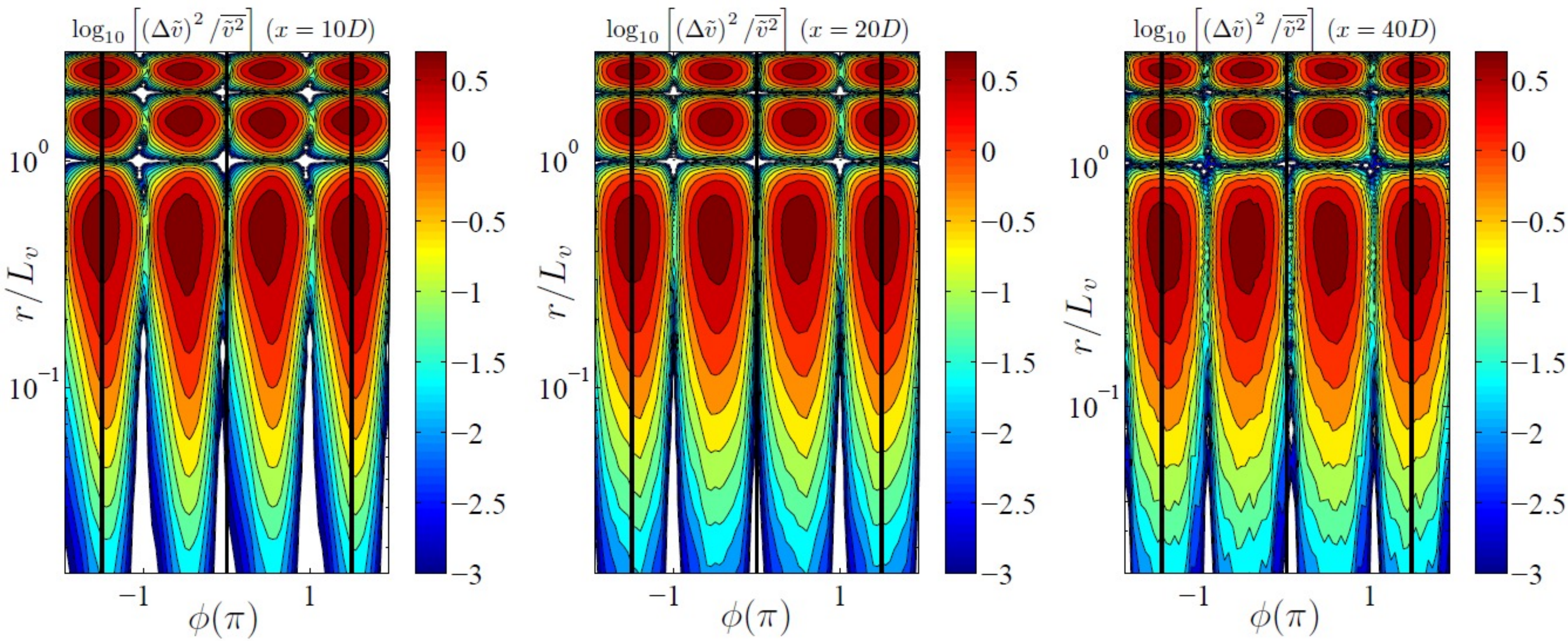
2. Assessment of Scale-by-Scale transport equations: Flows with CM



Less and less efect of CM for increasing downstream locations

2. Assessment of Scale-by-Scale transport equations: Flows with CM

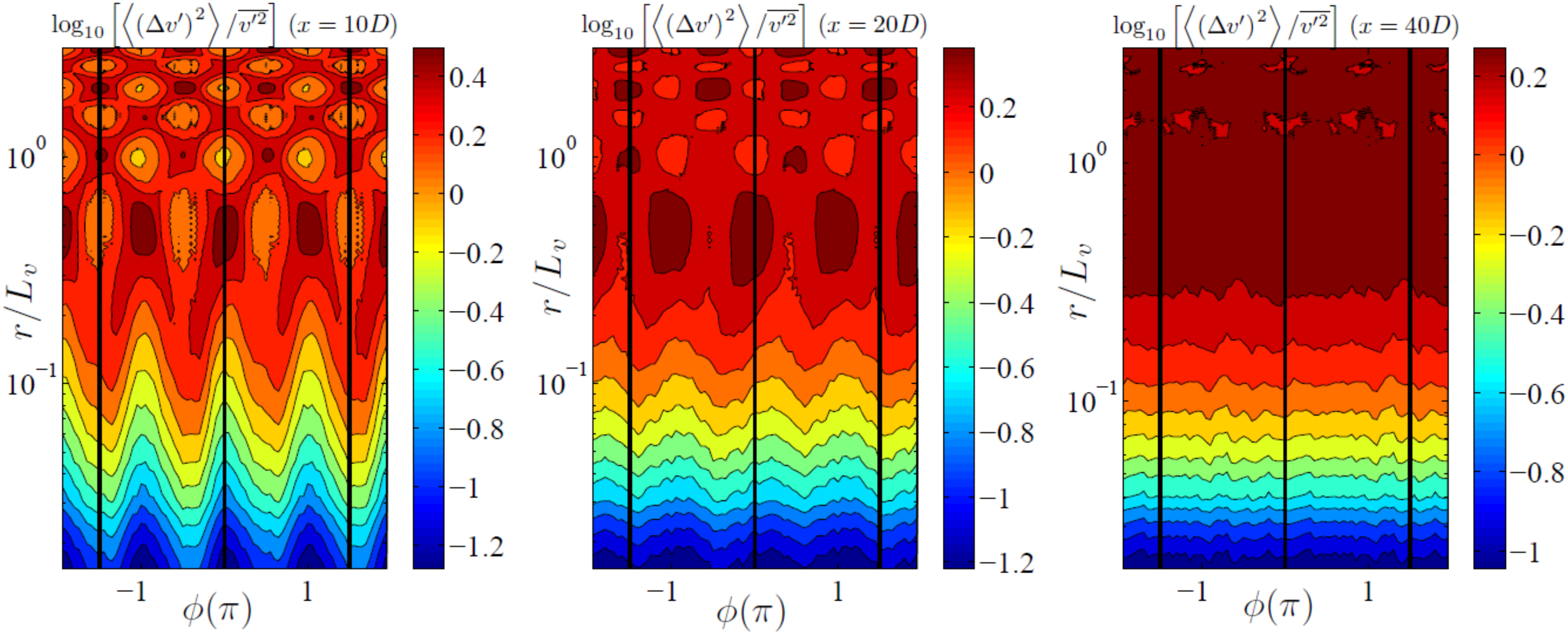
Phase-scale second-order structure functions for $\tilde{v}$



Persistance (relative) of CM for increasing downstream locations

2. Assessment of Scale-by-Scale transport equations: Flows with CM

Phase-scale second-order structure functions for v’



Less and less efect of CM for increasing downstream locations

2. Assessment of Scale-by-Scale transport equations: Flows with CM

1. As the distance downstream of the cylinder increases so does the scale at which the forcing due to the CM is perceptible.
   Phase-scale structure functions indicate that any scale r is correlated with that of the coherent shear whose effect is to locally enhance the energy at that scale.

2. In the presence of CM, isotropy may be satisfied from the smallest dissipative scale up to a scale which depends on the total strain. Its magnitude depends on the particular nature of the wake and the phase of the CM.

3. S-b-S energy budget equations which account for the CM have been derived using both general and isotropic forms. They indicate an additional forcing exerted by the CM on the random motion. Adequate agreement for the S-b-S energy budget of the RM.

Part of the K41 theory was rederived so it can be correctly applied to real, finite Reynolds numbers flows.

What about the SCALAR?

2. Methodology. Transport equations for a scalar

2.1. *Transport equations for* TT, $\overline{\tilde{\theta}\tilde{\theta}}$ *and* $\overline{\theta'\theta'}$

The starting point is the heat transport equation

$$\frac{\partial \Theta}{\partial t} + U_j \frac{\partial \Theta}{\partial x_j} = \kappa \frac{\partial}{\partial x_j}\frac{\partial \Theta}{\partial x_j} \quad \textbf{+ Source} \tag{2.1}$$

Following the triple decomposition of Reynolds & Hussain (1972), the velocity and temperature can be written as

$$\Theta = T + \tilde{\theta} + \theta' \tag{2.2a}$$

$$U_j = \overline{U}_j + \tilde{u}_j + u'_j \tag{2.2b}$$

Substituting (2.2) into (2.1), and then phase averaging, we obtain

$$\frac{\partial \tilde{\theta}}{\partial t} + \overline{U}_j \frac{\partial T}{\partial x_j} + \overline{U}_j \frac{\partial \tilde{\theta}}{\partial x_j} + \tilde{u}_j \frac{\partial T}{\partial x_j} + \tilde{u}_j \frac{\partial \tilde{\theta}}{\partial x_j} + \left\langle u'_j \frac{\partial \theta'}{\partial x_j} \right\rangle = \kappa \frac{\partial}{\partial x_j}\frac{\partial (T + \tilde{\theta})}{\partial x_j} \quad \textbf{+ Source} \tag{2.3}$$

The time average of (2.3) gives the equation for the mean temperature field:

$$\overline{U}_j \frac{\partial T}{\partial x_j} + \overline{\tilde{u}_j \frac{\partial \tilde{\theta}}{\partial x_j}} + \overline{u'_j \frac{\partial \theta'}{\partial x_j}} = \kappa \frac{\partial}{\partial x_j}\frac{\partial T}{\partial x_j} \tag{2.4}$$

2. Methodology. Transport equations

$$\frac{\overline{D\delta\tilde{\theta}^2}}{Dt} + \frac{\partial}{\partial X_\alpha}\left[\overline{\sum \tilde{u}_\alpha \delta\tilde{\theta}^2} + 2\overline{\left\langle \sum u'_\alpha \delta\theta' \right\rangle \delta\tilde{\theta}}\right] + \overline{2\delta\tilde{u}_\alpha\delta\tilde{\theta}\frac{\partial T}{\partial x_\alpha}}$$
$$-\overline{\left\langle \sum u'_\alpha \delta\theta' \right\rangle \frac{\partial}{\partial X_\alpha}\delta\tilde{\theta}} + \frac{\partial}{\partial r_\alpha}\overline{\delta\tilde{u}_\alpha\delta\tilde{\theta}^2} + \overline{2\delta\tilde{\theta}\frac{\partial}{\partial r_\alpha}\left\langle \delta u'_\alpha \delta\theta' \right\rangle}$$
$$-\kappa\left[\left(2\frac{\partial^2}{\partial r_\alpha^2} + \frac{1}{2}\frac{\partial^2}{\partial X_\alpha^2}\right)\overline{\delta\tilde{\theta}^2}\right] = -2\sum$$

+ Source 1

$$\frac{\overline{D\delta\theta'^2}}{Dt} + \frac{\partial}{\partial X_\alpha}\left[\overline{\sum u'_\alpha \delta\theta'^2} + \overline{\sum \tilde{u}_\alpha \left\langle \delta\theta'^2 \right\rangle}\right] + \overline{\left\langle \sum u'_\alpha \delta\theta' \right\rangle \frac{\partial}{\partial X_\alpha}\delta\tilde{\theta}}$$
$$+\overline{2\delta u'_\alpha \delta\theta'\frac{\partial T}{\partial x_\alpha}} + \frac{\partial}{\partial r_\alpha}\left(\overline{\left\langle \delta u'_\alpha \delta\theta'^2 \right\rangle} + \overline{\delta\tilde{u}_\alpha \left\langle \delta\theta'^2 \right\rangle}\right)$$
$$-\kappa\left[\left(2\frac{\partial^2}{\partial r_\alpha^2} + \frac{1}{2}\frac{\partial^2}{\partial X_\alpha^2}\right)\overline{\delta\theta'^2}\right] = -2\sum$$

+ Source 2

They indicate the additional forcing exerted by the CM on the random motion

Other terms are to be considered, accounting for the under-resolved scales! (ongoing work)

Operators allow for either 2D, or 3D turbulence to be tackled (ongoing work)

4. Conclusions

- Energy at each scale, put in the context of

first principles – from simple equations to the most complex, but closer to the reality

- interaction jet-coherent motions/waves/ eddies
- link to the energy injection on each atmospheric layer
- other effects are present: variable density, viscosity, differential diffusion
- floor to applications with data: large-scales ERA5, smaller scales: WRF….