SPECTRAL SIMULATIONS OF QUANTUM TURBULENCE USING THE GROSS-PITAEVSKII EQUATION

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We solve numerically the Gross-Pitaevskii (GP) equation:

$$i\frac{\partial}{\partial t}\psi(\mathbf{x},t) = \left(-\frac{1}{2}\nabla^2 + V(\mathbf{x}) + \beta|\psi(\mathbf{x},t)|^2 - \mathbf{\Omega}\cdot\mathbf{L}\right)\psi(\mathbf{x},t), \quad \text{in } \mathcal{D}\subset\mathbb{R}^3, \tag{1}$$

where β is the nonlinear interaction coefficient, $\mathbf{\Omega}$ the angular velocity and $\mathbf{L}$ the angular momentum: $\mathbf{L} = \mathbf{x}\times\mathbf{p}$, with $\mathbf{p} = -i\nabla$ the linear momentum. All variables are dimensionless, with the wave function ψ normalized to unity. We simulate the dynamics of Quantum Turbulence (QT) superflows for two distinct cases: *(i)* periodic box without trapping potential and rotation ($V = 0$, $\mathbf{\Omega} = 0$) and *(ii)* periodic box with confining (harmonic) potential ($V = \gamma\mathbf{x}^2/2$) and rotation around a major axis (*e. g.* $\mathbf{\Omega} = \Omega\mathbf{e}_z$). The former case corresponds to the classical setting [1, 2] GP-QT simulations of superflows (*e. g.* superfluid ^{4}He), while the latter corresponds to more recent settings of QT in Bose-Einstein condensates (BEC) [3]. Both cases are numerically simulated using the GPS (Gross-Pitaevskii Simulator) code [4]. GPS is a parallel MPI-OpenMP numerical code that is able to solve both real-time GP equation (1) and the corresponding imaginary-time equation using Fourier pseudo-spectral methods. Several time-integration schemes are available in the code: Strang splitting, Crank-Nicolson and relaxation method. The numerical code showed very good scaling properties on parallel supercomputers up to 96,000 cores. The highest grid resolution used in present computations was 1024^3.

We present below a typical result obtained for the case *(i)* of classical GP-QT simulations (periodic box with no trapping nor rotation). After validating the code using initial conditions based on Taylor-Green vortices [5] or random-phase fields [6], we tested a new type of initial condition based on randomly generated quantum vortex rings (see Figure 1). We will discuss in detail the characteristics of the QT generated with these three different initial conditions Simulations for the case *(ii)* of QT in rotating BEC will also be presented.

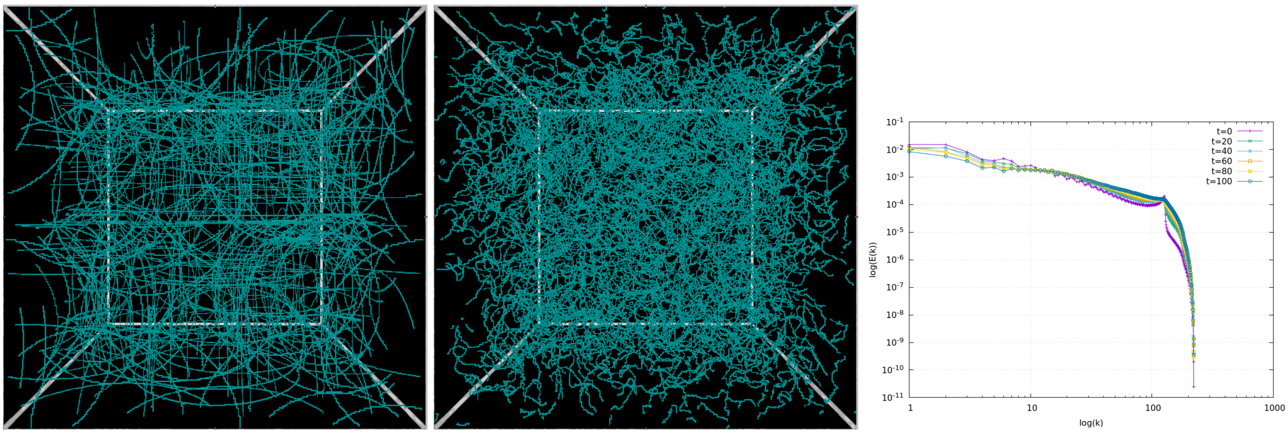

Figure 1. Spectral simulation of Quantum Turbulence in a period box using the Gross-Pitaevskii equation (without trapping potential). From left to right: initial condition with randomly distributed vortex rings, final turbulent state and time evolution of the kinetic-energy spectrum. Grid resolution: 512^3.

References

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