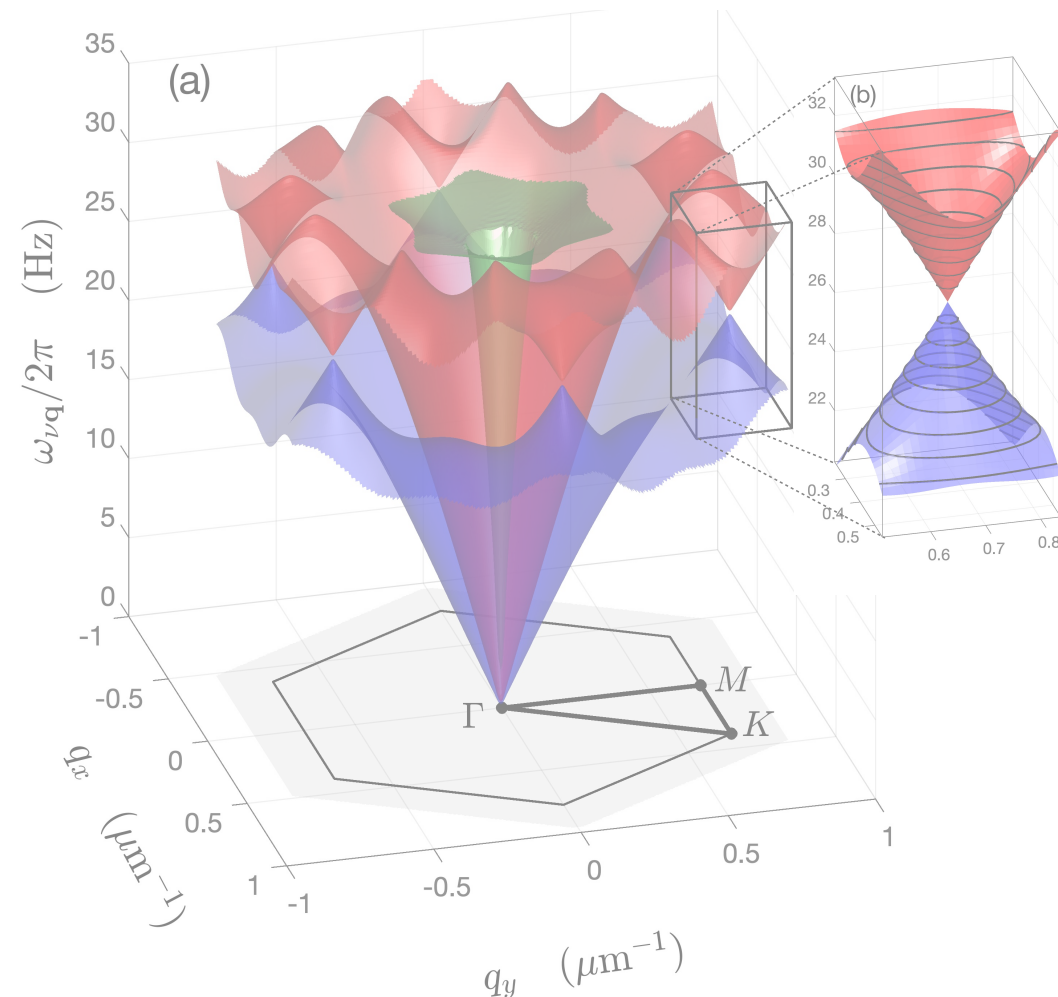
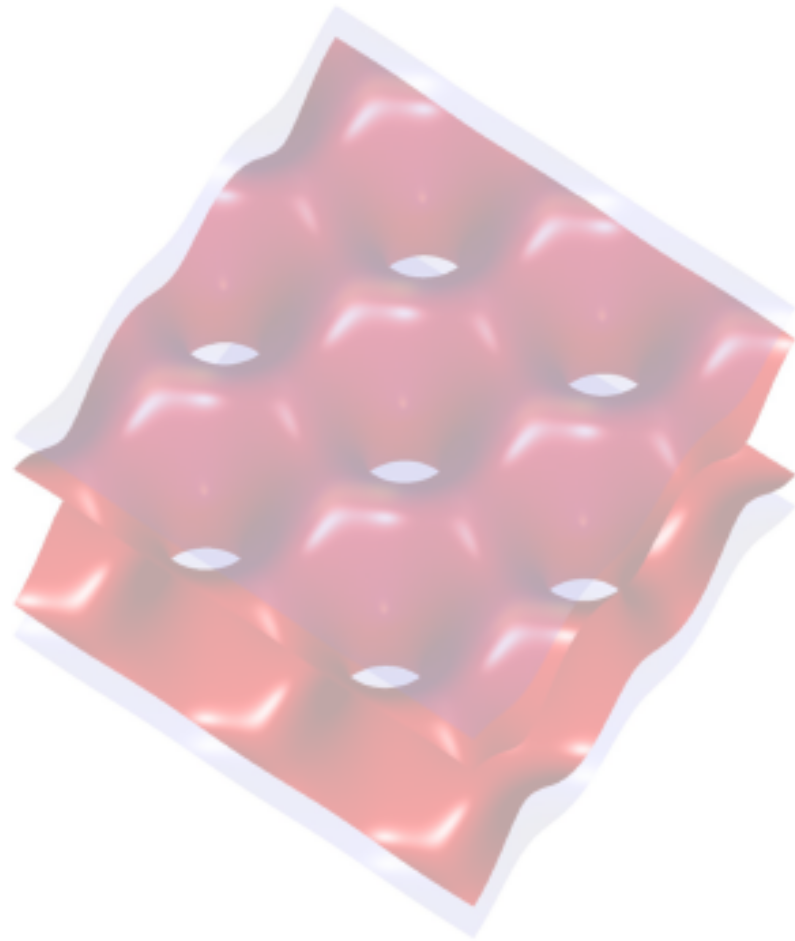


Supersolidity in Dipolar Bose-Einstein condensates



P Blair Blakie

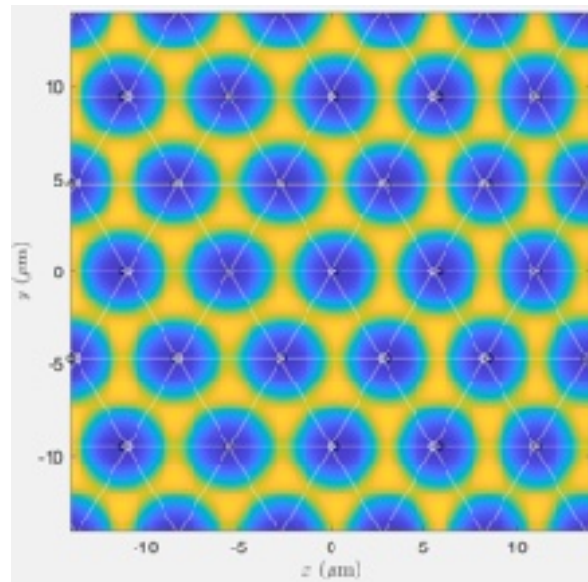
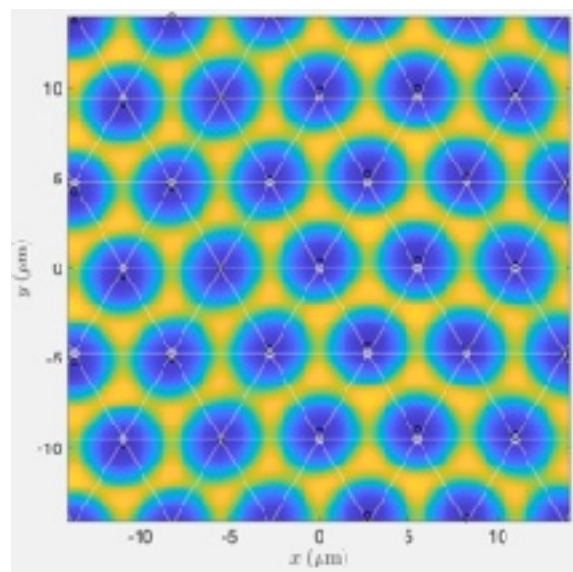
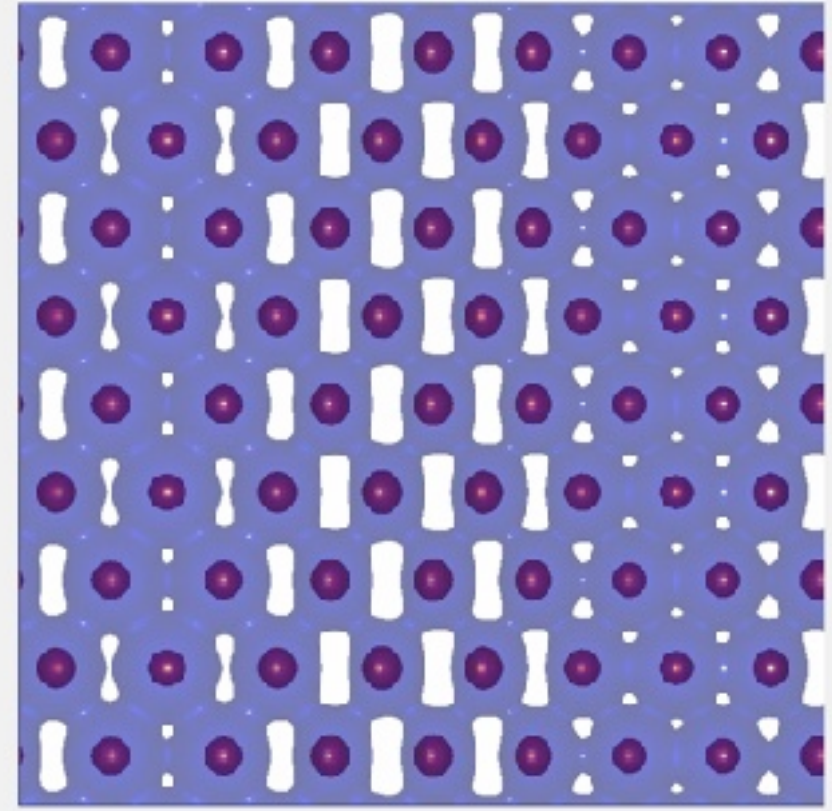
Department of Physics, University of Otago, Dunedin, New Zealand

AIMS - Athens 2026

Advances in mathematical modelling and numerical simulation of superfluids

Outline

- Introduction to Supersolids
- Dipolar systems and experiments
- 2D Dipolar supersolids – supersolid **hydrodynamics and sound**
- Tilt and anisotropic supersolids
- Two component dipolar supersolids
- Outlook



Key people involved with our work:

Danny Baillie, Reuben Cook, Au-Chen Lee, Lily Platt, Elena Poli, Ben Ripley, Rene Rohrs

Francesca Ferlaino, Lauriane Chomaz, Russell Bisset

What is a supersolid?

A **supersolid** is a material that has two conflicting properties

1. **Rigid structure of a solid** - breaks translational symmetry
2. **Frictionless flow of a superfluid** - breaks U(1) gauge symmetry

Early theoretical work:

- A. F. Andreev and I. M. Lifshitz, *Quantum Theory of Defects in Crystals*, JETP, **29**, 1107 (1969).
- G. V. Chester, *Speculations on Bose-Einstein Condensation and Quantum Crystals*, PRA, **2**, 256 (1970).
- A. J. Leggett, *Can a Solid Be “Superfluid”?*, PRL **25**, 1543 (1970)

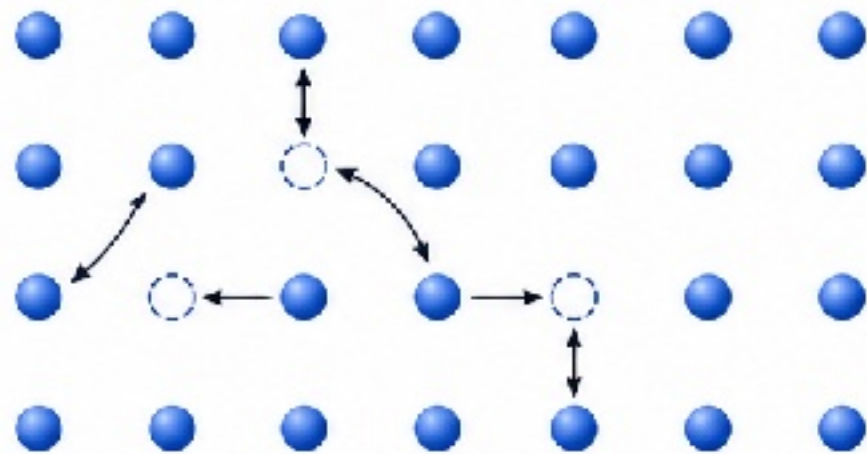
Solid He-4 experiments - Anomalous moment of inertia:

- E. Kim and M. H. W. Chan, *Probable observation of a supersolid helium phase*, Nature **427**, 225 (2004).
- D. Y. Kim and M. H. W. Chan, *Absence of Supersolidity in Solid Helium in Porous Vycor Glass*, PRL **109**, 155301 (2012).

Two limiting routes to supersolidity

1. Solid that is superfluid (defects tunnel)

Crystalline order with superfluid defects that can move (tunnel)



Key idea:

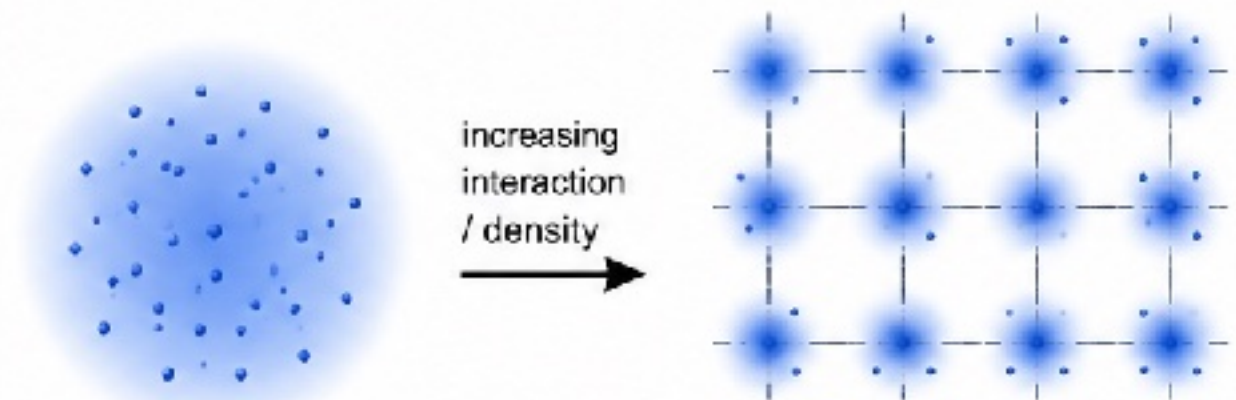
Long-range crystalline order is preserved, while defects (vacancies / interstitials) delocalize and carry superfluid flow.

- particle
- vacancy / interstitial
- ↔ tunneling of defects

Examples: ⁴He supersolid (defect BEC),
dipolar / quantum solids with mobile vacancies

2. Superfluid that becomes solid (density modulates)

Uniform superfluid develops density modulations (crystalline order)



Key idea:

A uniform superfluid (global phase coherence) spontaneously develops periodic density modulations, forming a solid while remaining superfluid.

- high density (lattice site)
- particles
- emergent lattice

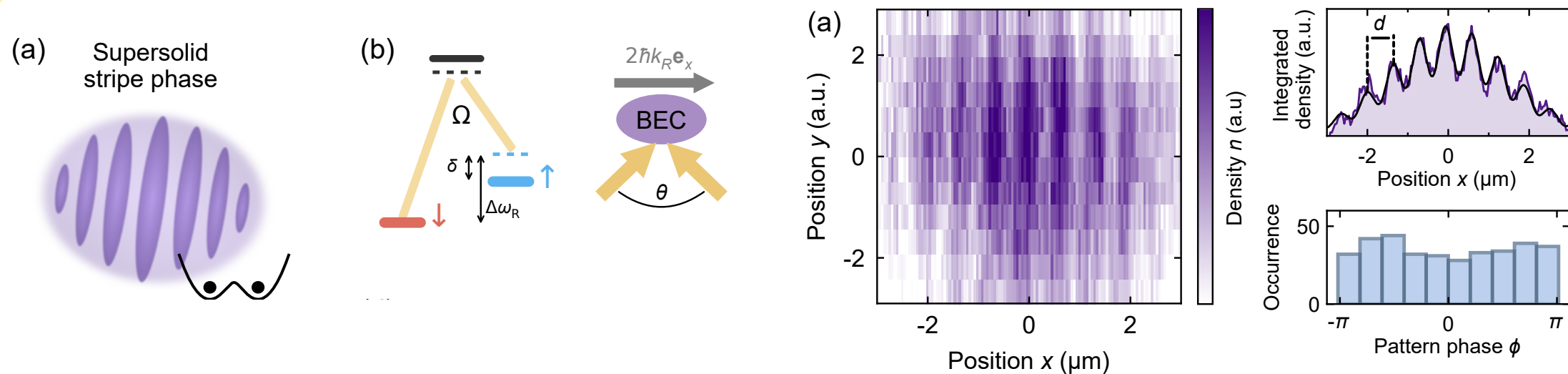
Examples: Soft-core bosons in optical lattices,
Rydberg-dressed gases, extended interactions

Both routes break continuous translation symmetry (density wave) while preserving phase coherence → supersolid.

Examples of experimental results

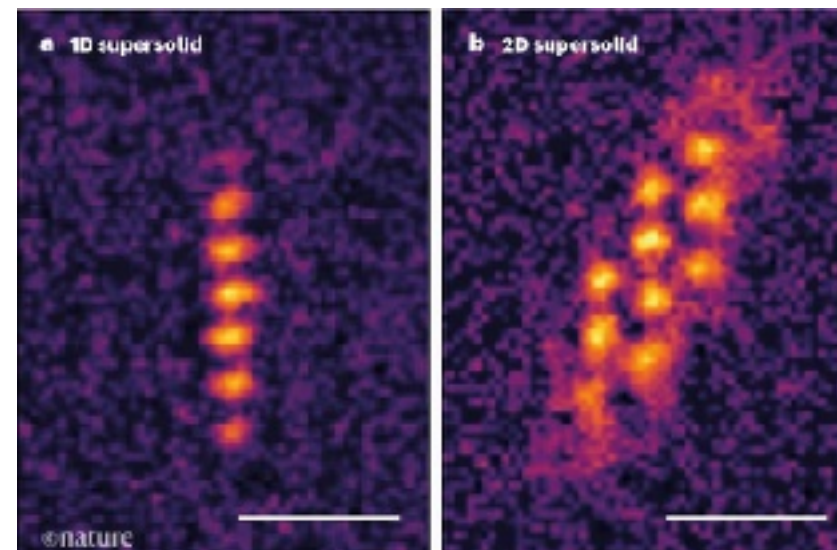
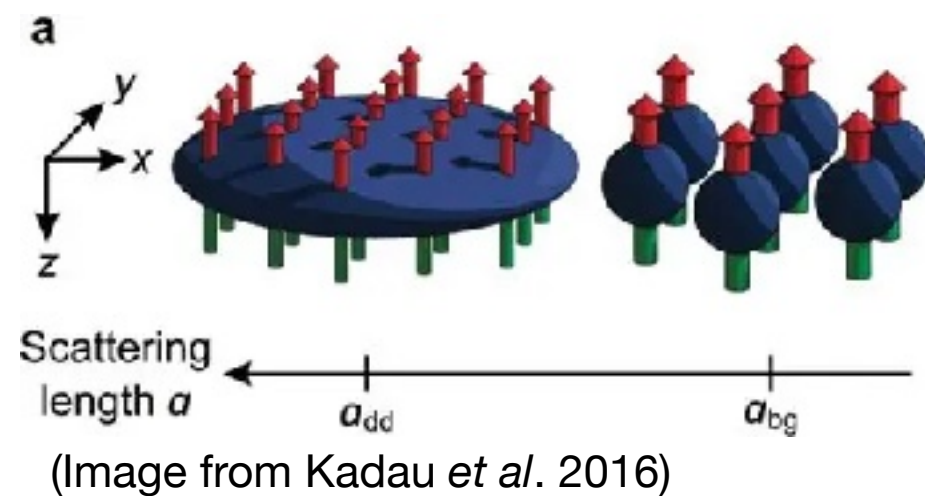
Spin-orbit coupled BEC

(Barcelona group, Science 2026)



Dipolar BECs

(Innsbruck group, Nature 2021)



Can a Solid Be "Superfluid"?

Bounds for a supersolid

Upper bound (crude):

$$\rho_s^{(e)} \leq \rho_{ub}^{(e)} = \left(\frac{1}{\langle \rho \rangle} \right) \langle \rho(r) \rangle_s$$

where $\langle \dots \rangle_s$ denotes the average along the flow direction s .



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Can a Solid Be "Superfluid"?

A. J. Leggett

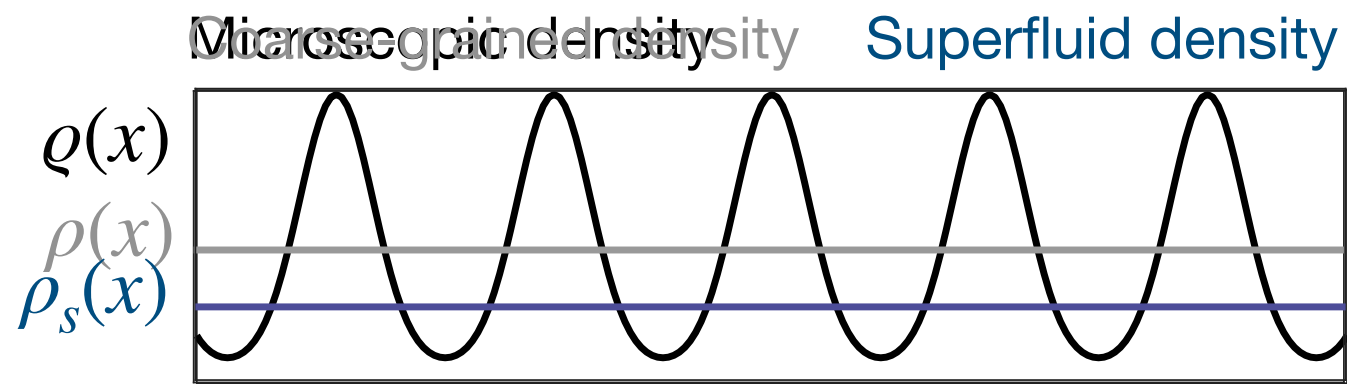
School of Mathematical and Physical Sciences, University of Sussex, Falmer, Brighton, Sussex, England

(Received 15 September 1970)

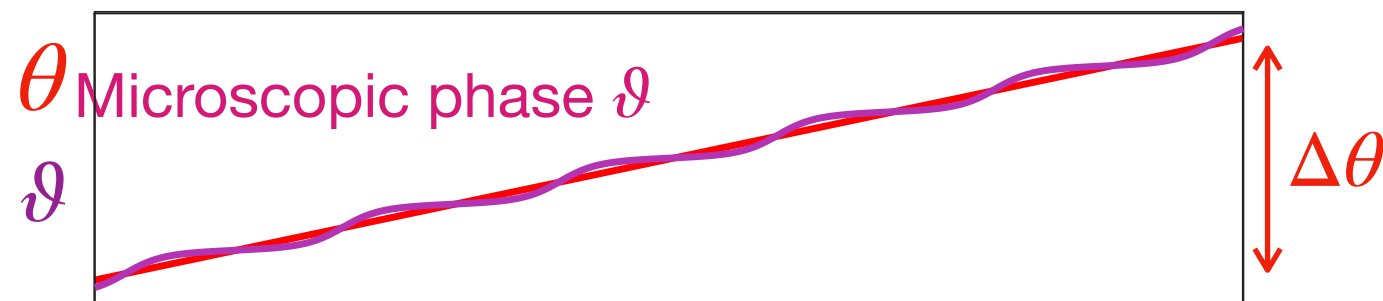
It is suggested that the property of nonclassical rotational inertia possessed by superfluid liquid helium may be shared by some solids. In particular, nonclassical rotational inertia very probably occurs if the solid is Bose-condensed as recently proposed by Chester. Anomalous macroscopic effects are then predicted. However, the associated superfluid fraction is shown to be very small (probably $\lesssim 10^{-4}$) even at $T=0$, so that these effects could well have been missed. Direct tests are proposed.

PRL 1970

Why is the superfluid fraction not 100% at $T = 0$?



Twist (macroscopic) phase θ at the boundaries and solve for stationary state

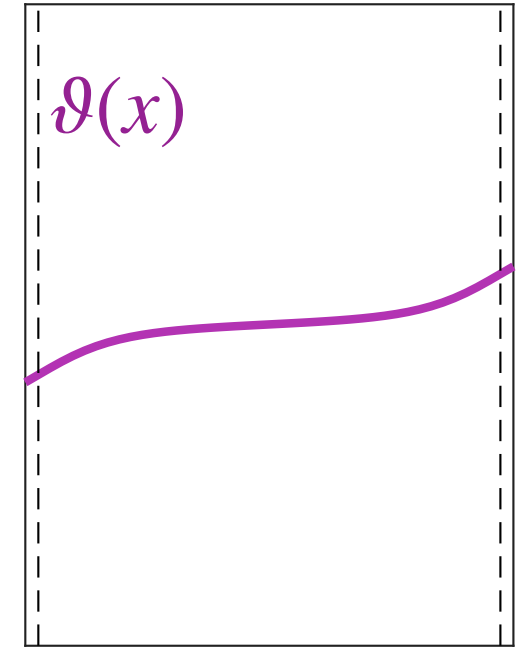
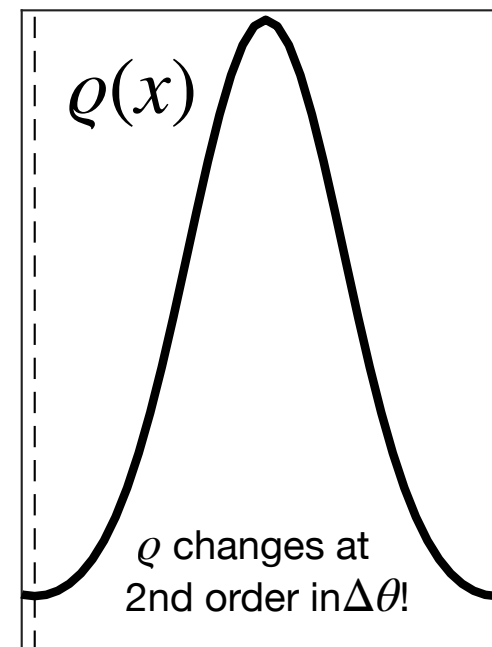


Phase steps reveal reduced superfluid fraction

In 1D we have:

$$J_x = \text{const} \longrightarrow v(x) = J/q(x)$$

Focus on a single lattice site



? Current ?

$$J_x = v(x)q(x)$$

Constant: b/c ψ stationary!

$$\partial_t q = -\nabla \cdot \mathbf{J}$$

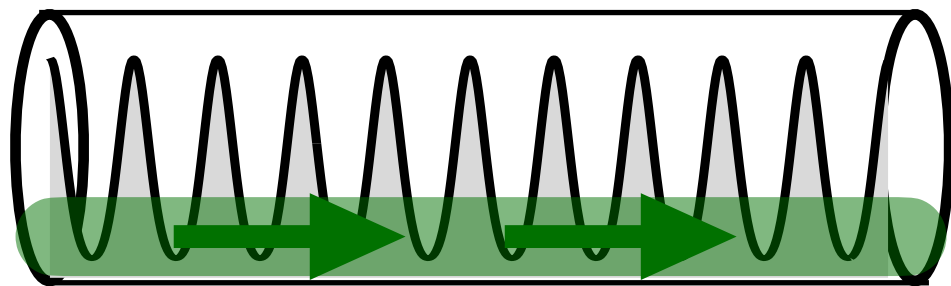
Local velocity

$$v(x) = \frac{\hbar}{m} \partial_x \vartheta$$

Supersolid in a pipe

Twist phase $v_s = \frac{\hbar}{m} \frac{\Delta\theta}{L}$

Stationary crystal



Flowing superfluid J_x

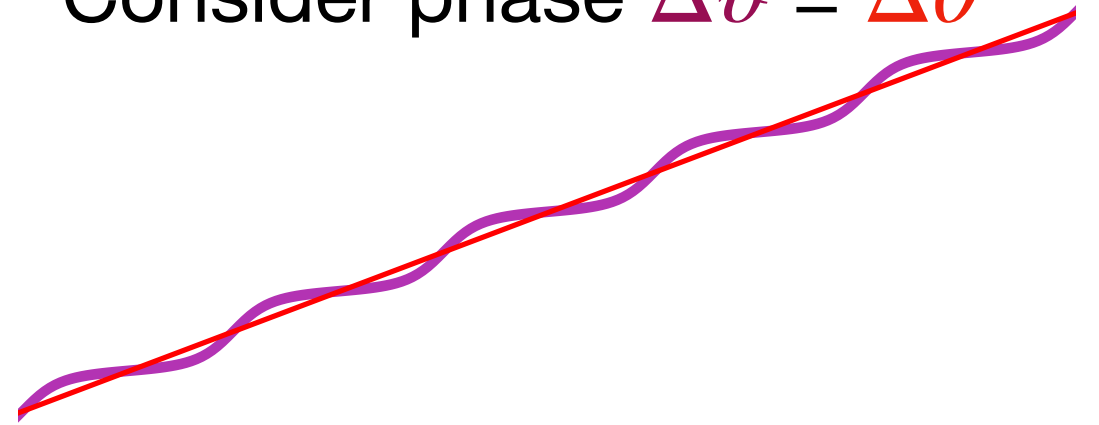
Leggett superfluid fraction

$$\rho_s = \left\langle \frac{1}{\rho(x)} \right\rangle^{-1}$$

Leggett PRL 1970

Upper bound ($T=0$). Exact for 1D meanfield

Consider phase $\Delta\vartheta = \Delta\theta$



$$\Delta\vartheta \propto \int v(x) dx = J_x \int \frac{1}{\rho(x)} dx$$

Equate to $\Delta\theta \propto v_s L$ Spatial ave

$$v_s = J_x \left\langle \frac{1}{\rho(x)} \right\rangle \quad \langle . \rangle = \frac{1}{L} \int dx.$$

Superfluid relationship

$$J_x = \rho_s v_s$$

Expts: 1D optical lattices LKB and JQI;
1D supersolids by Pisa

Superfluidity in Higher dimensions

Phase twist: $\psi_0 = \sqrt{\varrho(\mathbf{x})} \longrightarrow \sqrt{\varrho(\mathbf{x})} e^{i\phi_1(\mathbf{x}) + i\frac{m}{\hbar} \mathbf{v}_s \cdot \mathbf{x}}$ ($\lim \mathbf{v}_s \rightarrow 0$)

The stationary condition:

$$\nabla \cdot \mathbf{J} = 0 \longrightarrow \frac{\hbar}{m} \nabla \cdot \left(\varrho(\mathbf{x}) \nabla \phi_1(\mathbf{x}) \right) = - \mathbf{v}_s \cdot \nabla \varrho(\mathbf{x})$$

weighted Poisson equation

Saslow, PRL 1976

Coarse-grain to extract superfluidity:

$$\bar{\mathbf{J}} = \frac{1}{V} \int d\mathbf{x} \varrho(\mathbf{x}) \left[\mathbf{v}_s + \frac{\hbar}{m} \nabla \phi_1(\mathbf{x}) \right] \longrightarrow \bar{J}_i = \sum_j \rho_{ij}^{(s)} v_{s,j}.$$

Superfluid **tensor**: $\rho_{ij}^{(s)} = \bar{\varrho} \delta_{ij} + \frac{\hbar}{mV} \int d\mathbf{x} \varrho(\mathbf{x}) \partial_i \left(\frac{\partial \phi_1}{\partial v_{sj}} \right),$

Leggett bound(s) work but are usually not tight.

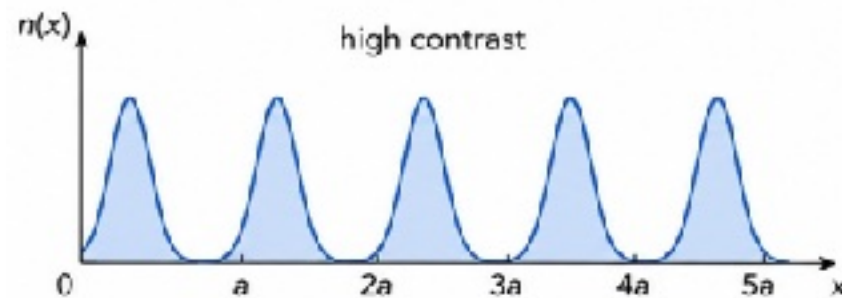
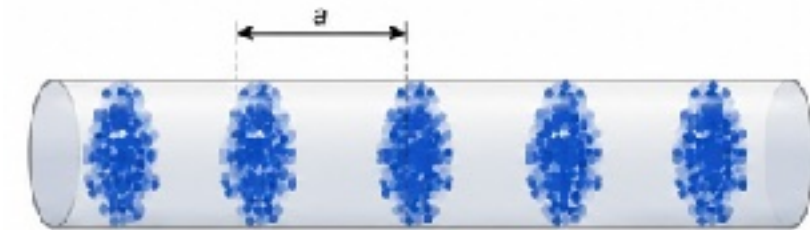
Expts: with 2D optical lattices LKB (Rabec PRL 2026)

Key challenge

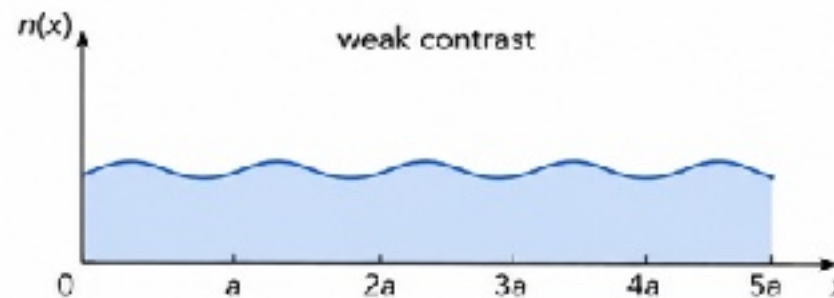
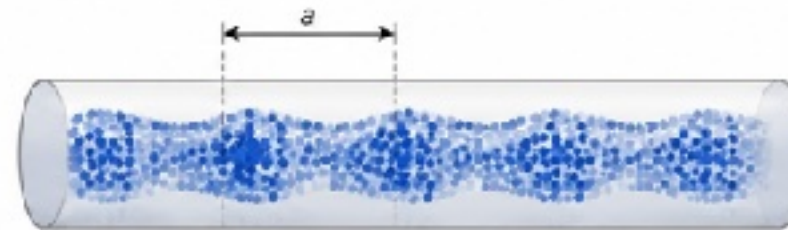


Leggett upper
bound $\rho_s = \left\langle \frac{1}{\varrho(x)} \right\rangle^{-1}$

$$f_s = \rho_s / \rho$$



$f_s \rightarrow 0$ Independent droplets



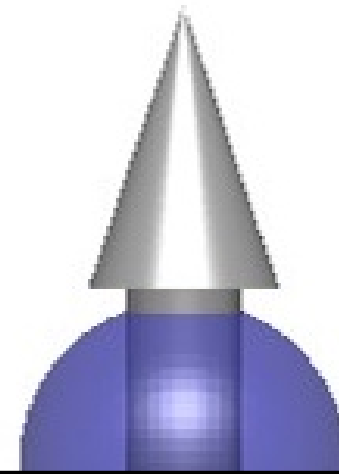
$f_s \rightarrow O(1)$ Supersolid

- Ideally, want: low contrast density modulation or a highly connected crystal.

Dipolar systems & Experiments

Magnetic dipole-dipole interactions

μ



μ

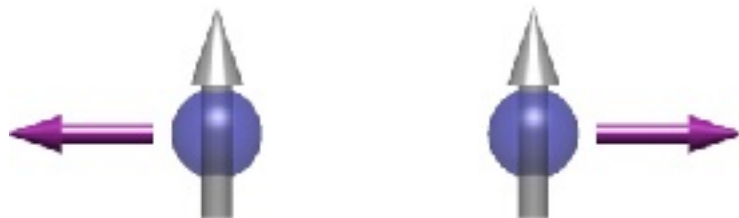


Magnetic DDI

$$U_{\text{dd}}(\mathbf{r}) = \frac{\mu_0 \mu^2}{4\pi} \frac{1 - 3(\hat{\mu} \cdot \hat{\mathbf{r}})^2}{r^3}$$

Anisotropic

Repulsive



Attractive



Longrange

$$\sim \frac{1}{r^3}$$

Tuneable two body interactions

Feshbach resonance - reducing g_s

Two-body interactions

$$U(\mathbf{r}) = g_s \delta(\mathbf{r}) + \frac{g_{dd}}{4\pi} \frac{1 - 3 \cos^2 \theta}{r^3}$$

$$g_s = \frac{4\pi a_s \hbar^2}{m}$$

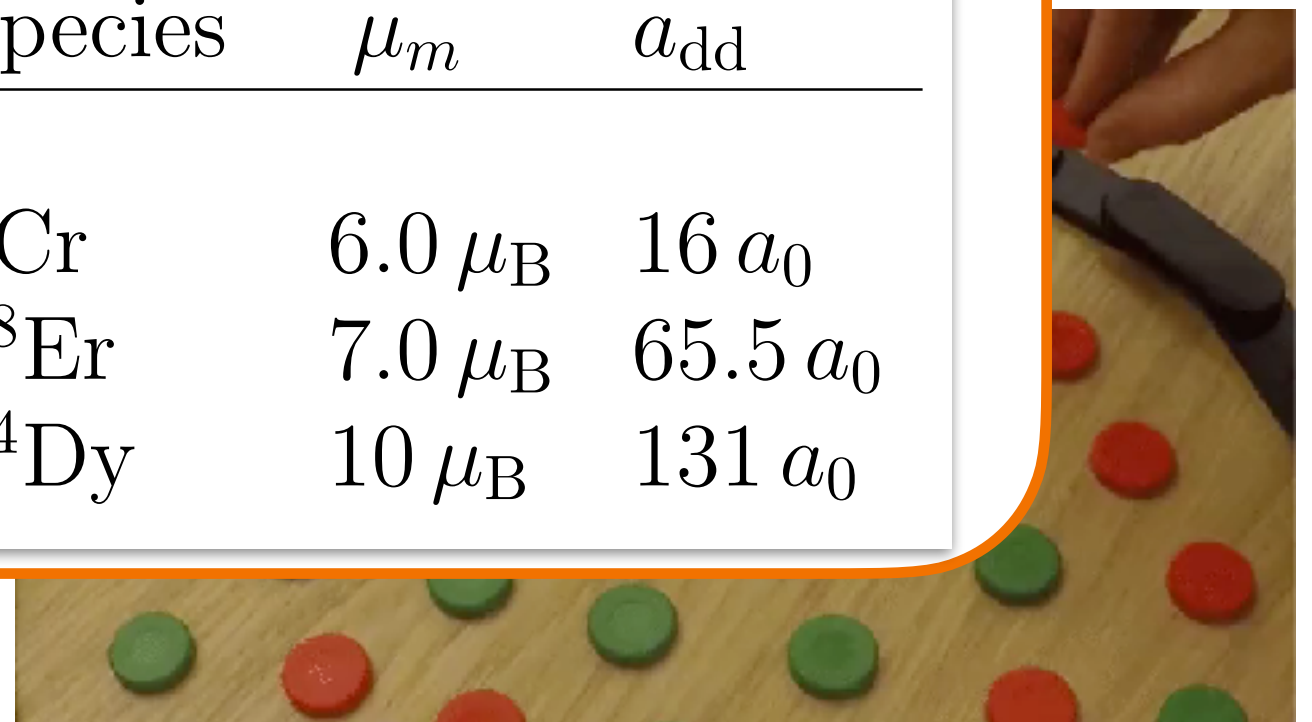
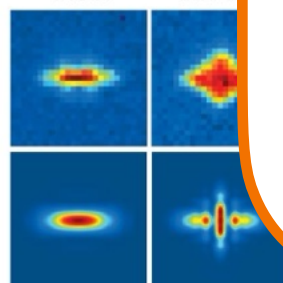
$$g_{dd} = \frac{4\pi a_{dd} \hbar^2}{m}$$

Controllable with
magnetic Feshbach
resonances

Species	μ_m	a_{dd}
^{52}Cr	$6.0 \mu_B$	$16 a_0$
^{168}Er	$7.0 \mu_B$	$65.5 a_0$
^{164}Dy	$10 \mu_B$	$131 a_0$

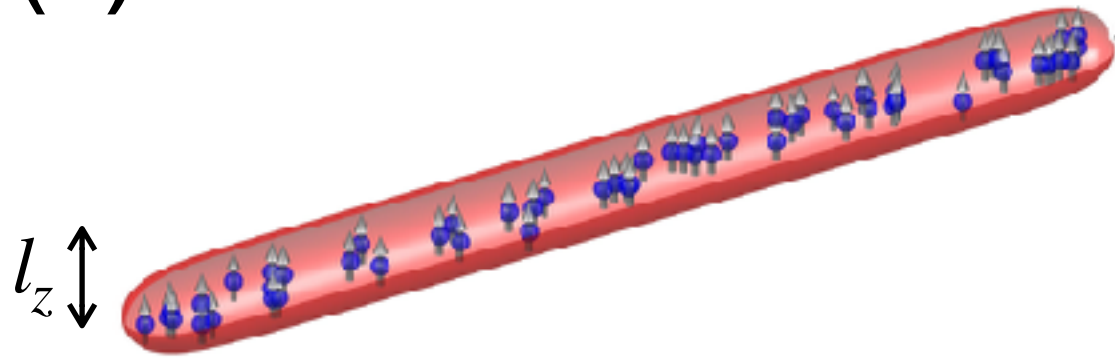
Collapse dy

0 ms 0.1 ms



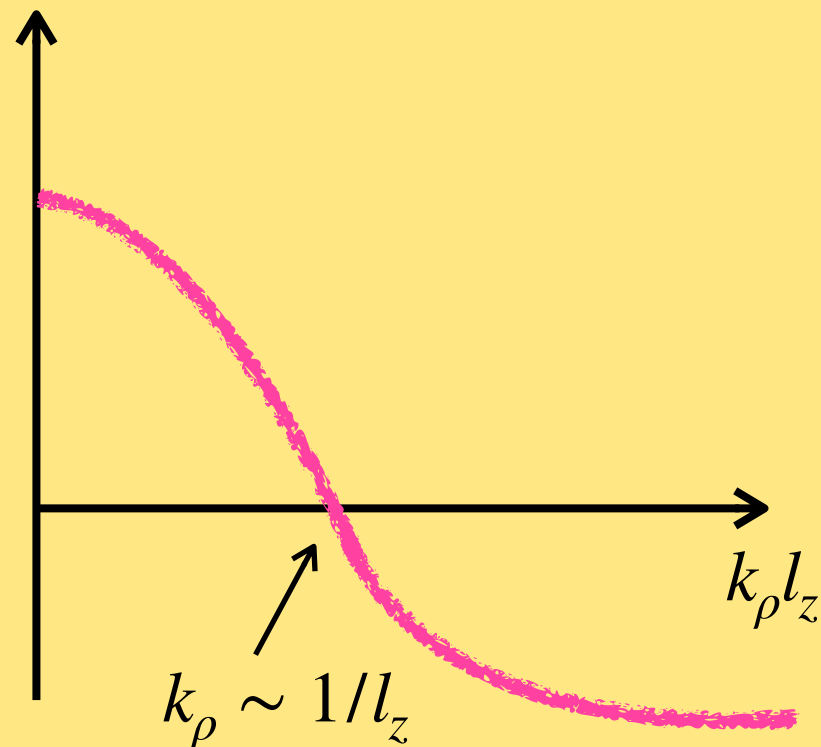
Key ingredient for supersolidity:

(1) Confinement along dipole direction



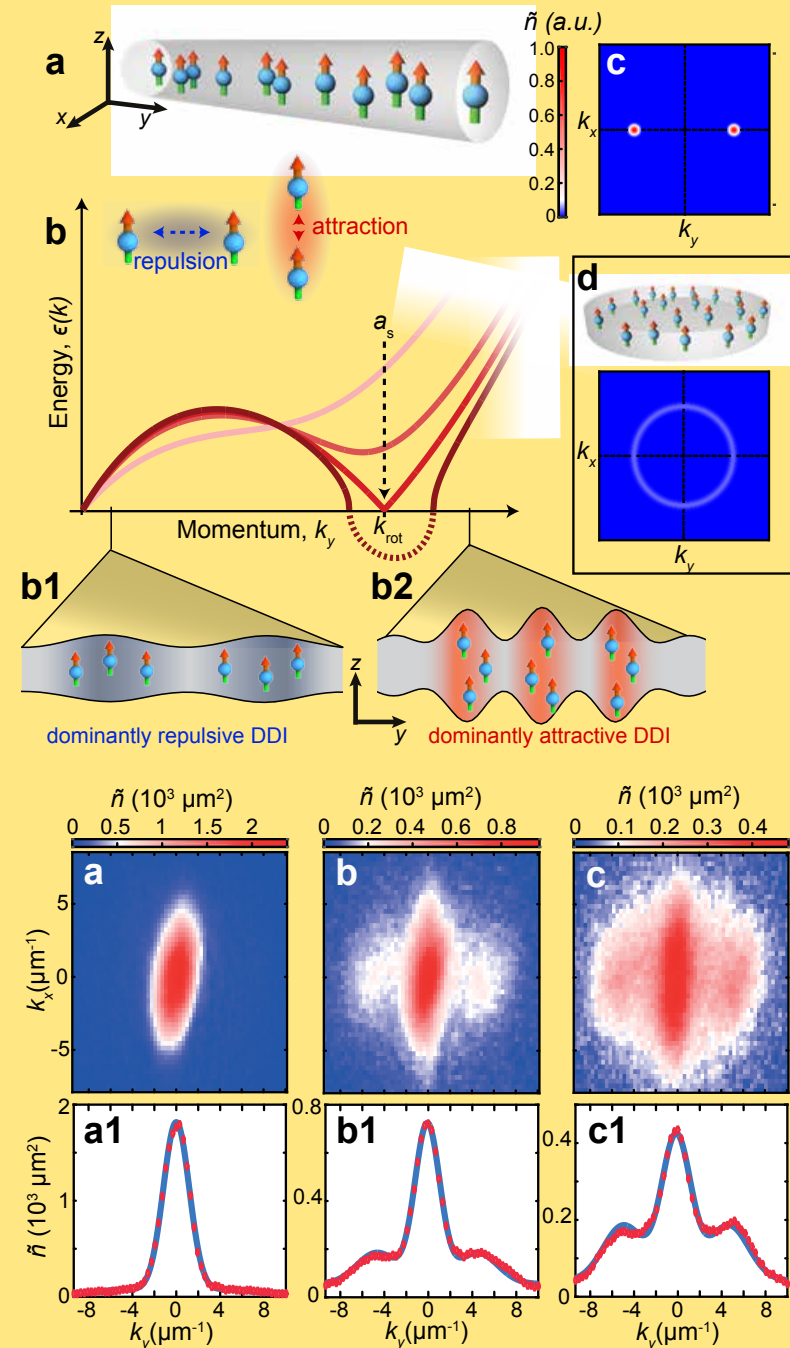
$\Rightarrow k_\rho$ -dependent interaction

$$\tilde{U}_{DDI}(k_\rho)$$



E.g., see: Cai *et al.*, PRA (2010); Baillie *et al.*, NJP (2015); Blakie *et al.*, CTP (2020);

Favours spatial structure, e.g. rotons



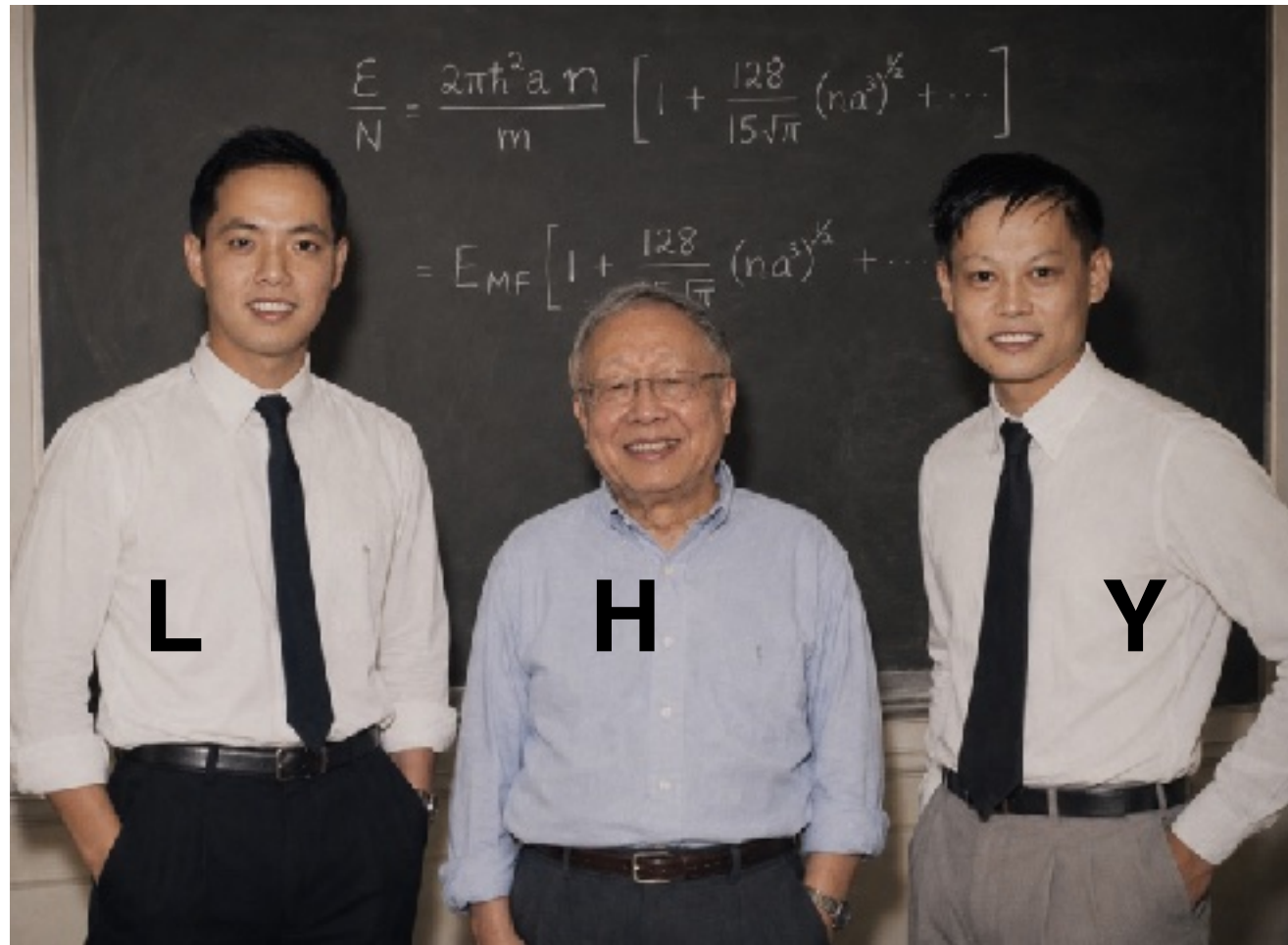
Chomaz *et al.*, Nature Physics (2018)

Key ingredient for supersolidity:

(2) Beyond-meanfield effects

Collapse stabilization

Beyond meanfield physics (Quantum fluctuations)

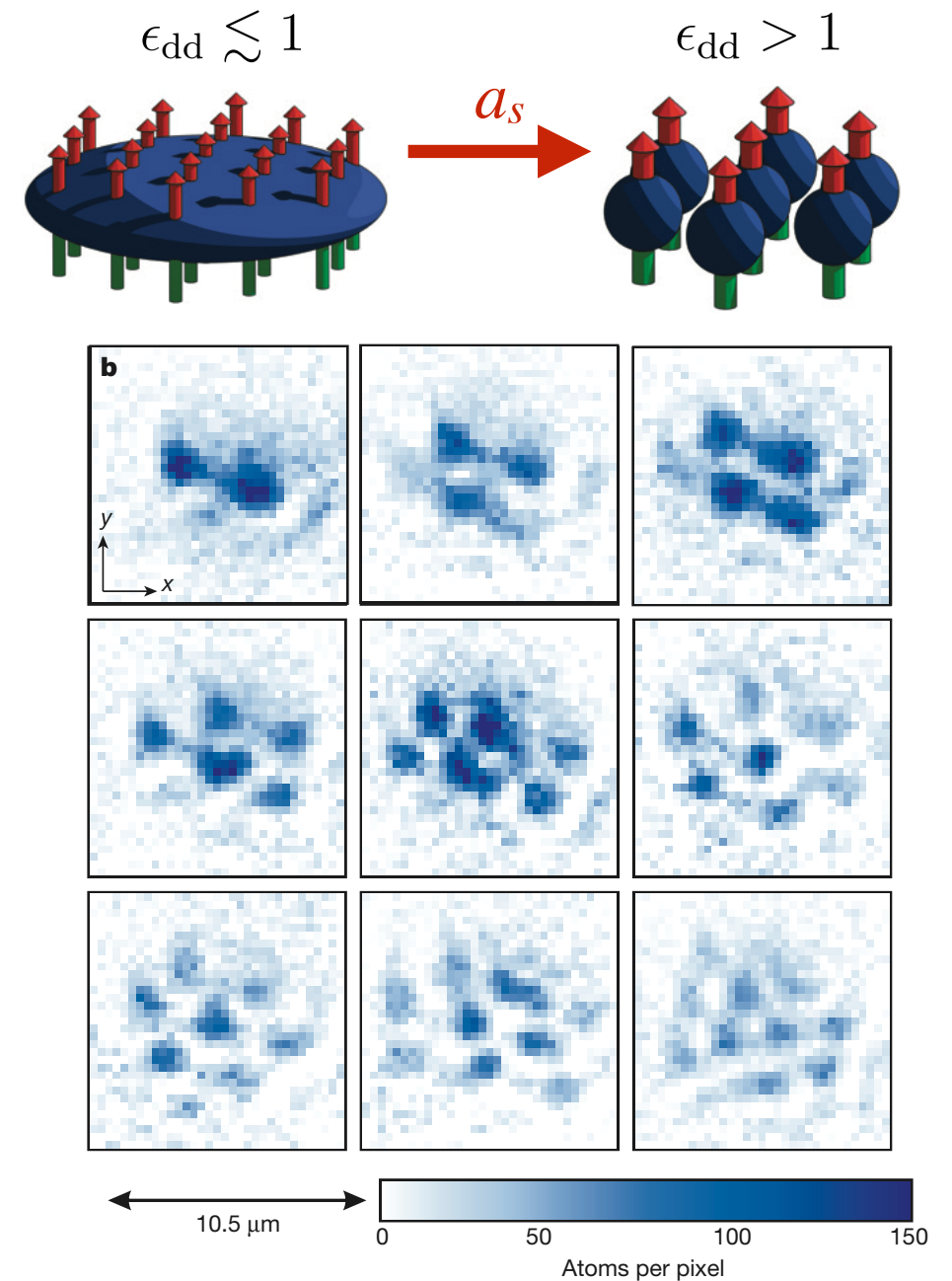


T.D. Lee and C.N. Yang, (1957).; T.D. Lee, K. Huang, and C.N. Yang, (1957); Lima and Pelster, (2011)

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H. Kadau, et al., Nature **530**, 194 (2016).

Experiment

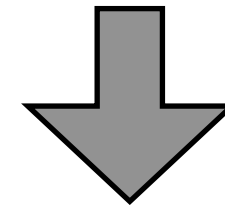
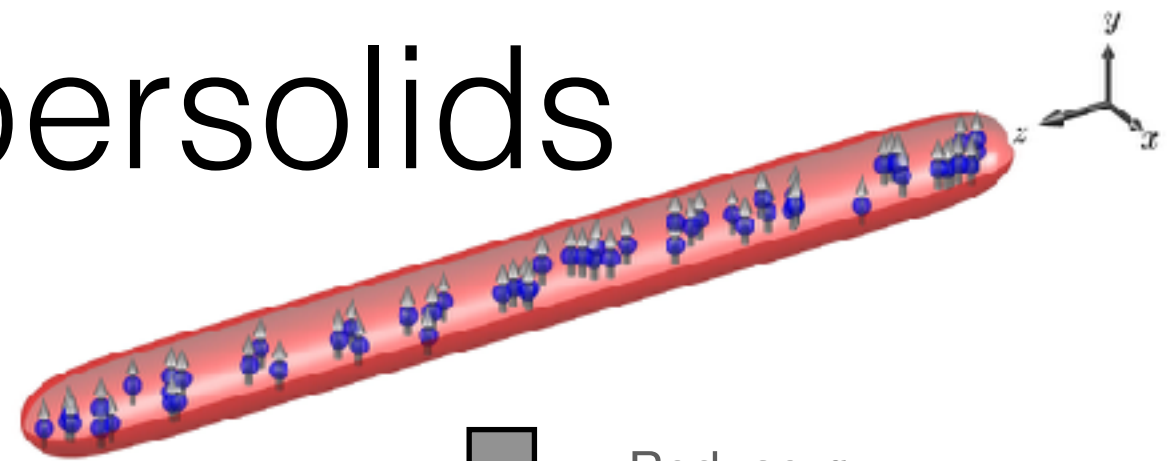


Droplet crystal (not a supersolid)

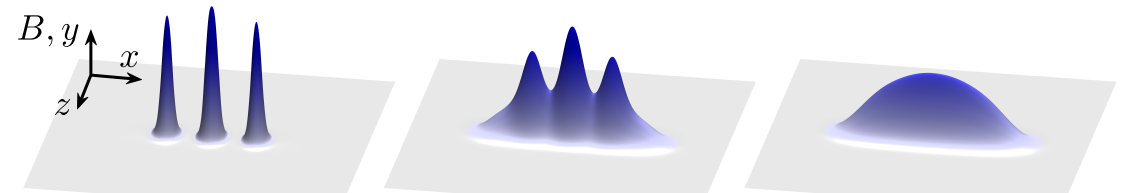
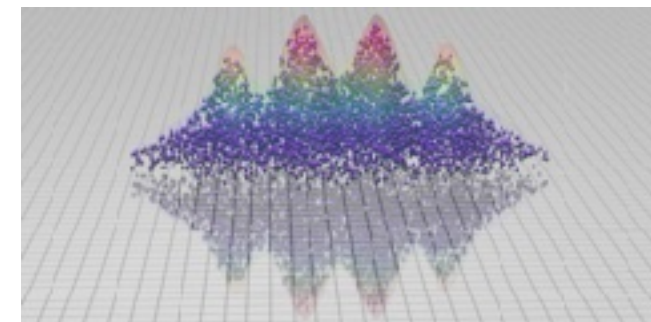
Dipolar Supersolids

System:

- Axially-elongated dipolar BEC
- Dipoles along a tightly confined direction

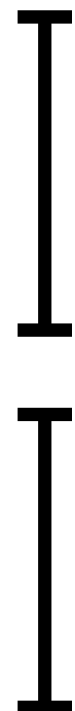


Reduce a_s
Transition to supersolid



Three Experiments

- Tanzi *et al.* PRL **122** (2019)
- Bottcher *et al.*, PRX **9** (2019)
- Chomaz *et al.*, PRX **9** (2019)
- Natale *et al.*, PRL **123** (2019)
- Tanzi *et al.*, Nature **574** (2019)
- Guo *et al.*, Nature **574** (2019)

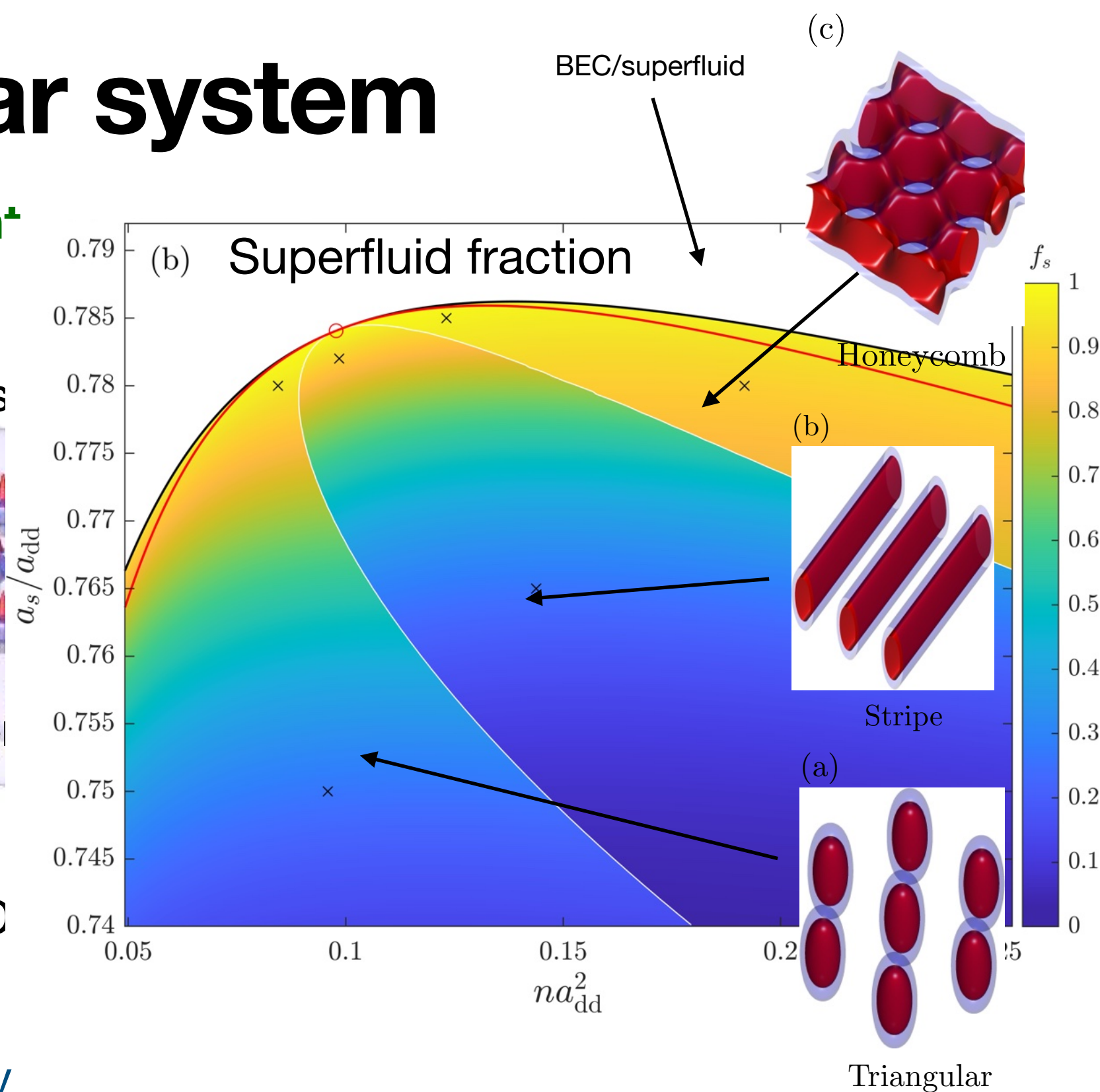
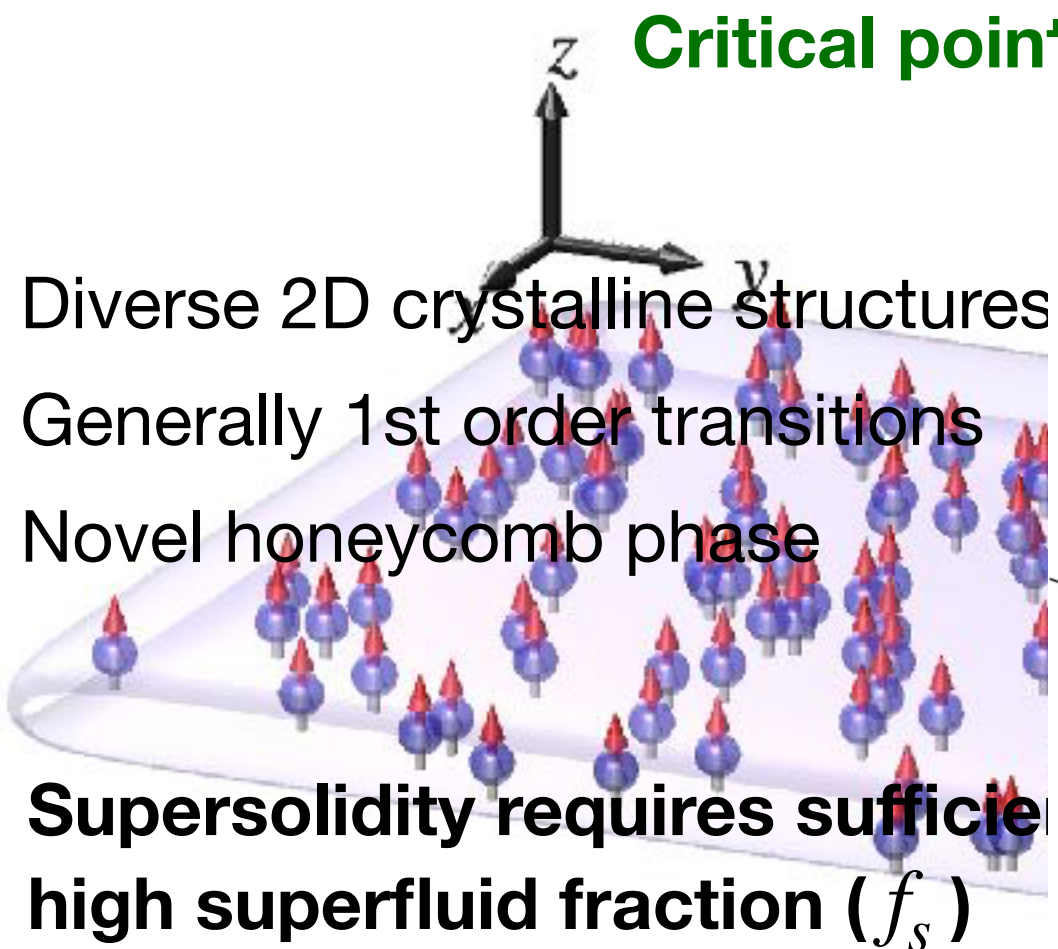


Observation of supersolid state

* crystalline order (3 -5 unit cells)

Observation of supersolid excitations

Planar dipolar system



Axially confined (72.4 Hz) ^{164}D

Two control parameters:

ρ - average (areal) density

a_s - s-wave contact interaction

Thermodynamic limit: Zhang *et al.* PRL (2019); Ripley *et al.*, PRA (2023);

Pancake trapped case: Poli *et al.*, PRA (2021); Hertkorn *et al.*, PRR (2021); Zhang *et al.*, PRA (2021)

Excitations

- Excitations: waves on the supersolid.

- Cf. Hydrodynamics for a superfluid
- $$\hbar^2 \left(\frac{1}{\alpha_{\rho\rho}} \frac{\partial^2}{\partial t^2} - \frac{\rho_s}{m} \nabla^2 \right) \phi = 0$$
- (Bulk modulus) compressibility
- superfluid density
- Speed of sound $c_s = \sqrt{\rho_s \alpha_{\rho\rho} / m}$

**Can we similarly
analyze a supersolid?**

**Can we analyze the excitations beyond large
scale calculations?**

One **transverse** sound branch

Two **longitudinal** sound branches

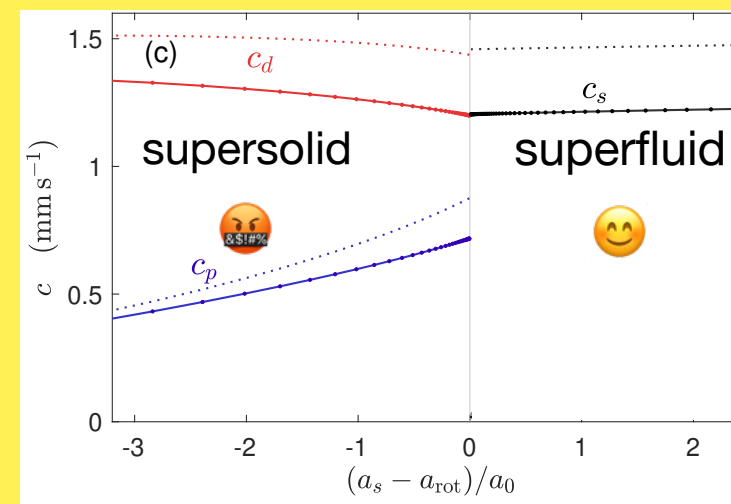
Context

1. Supersolid hydrodynamics and homogenized theories:

Andreev, Saslow, Son, Dorsey, Rica, Zwerger,.....

2. Microscopic calculations; Meanfield, QMC of soft-core, dipolar BECs,....:

Rica, Pohl, Boninsegni, Ancilotto,.....



Our work: **microscopic comparison
establishing supersolid hydrodynamics** for
1D soft-core and dipolar (2024); 2D dipolar
(2025); 2D anisotropic dipolar (2026);

* DKK Lee 2D soft-core (2024); Recati/Stringari:
dipolar supersolid dynamics comparisons (2024).

Calculations

To determine wavefunction $\psi(\mathbf{x})$ which minimises the energy density:

Planar kinetic energy	Axial single particle energy	2-body interactions
$T_v = \frac{(-i\hbar\nabla_\perp + m\mathbf{v})^2}{2m}$	$H_z = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial z^2} + \frac{1}{2}m\omega_z^2 z^2$	$\Phi(\mathbf{x}) = \int d\mathbf{x}' U(\mathbf{x} - \mathbf{x}') \psi(\mathbf{x}') ^2$

$$\mathcal{E}(\rho, \mathbf{a}_1, \mathbf{a}_2, \mathbf{v}) = \frac{1}{A} \int_{\text{uc}} d\mathbf{x} \psi^* \left(T_v + H_z + \frac{1}{2}\Phi + \frac{2}{5}\gamma_{QF} |\psi|^3 \right) \psi$$

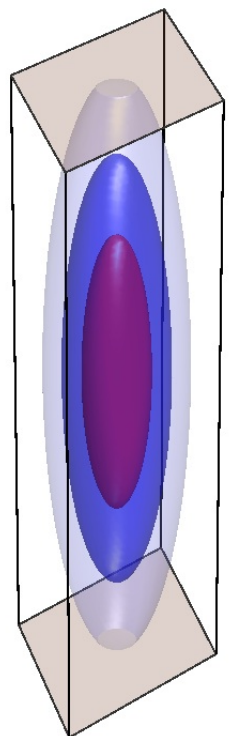
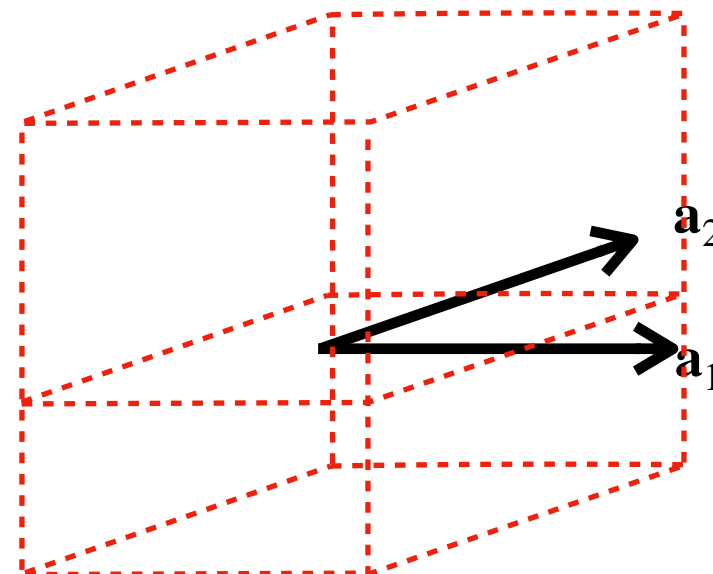
- A unit cell (uc) area with planar lattice vectors $\{\mathbf{a}_1, \mathbf{a}_2\}$, $A = |\mathbf{a}_1 \times \mathbf{a}_2|$
- $\mathbf{v}$ is the superfluid velocity, imposing twisted periodic boundary conditions $\psi(\mathbf{x} + \mathbf{a}_j) = \psi(\mathbf{x}) e^{i\mathbf{v} \cdot \mathbf{a}_j}$

- ρ specified ave. areal density:

$$\int_{\text{uc}} d\mathbf{x} |\psi|^2 = \rho A$$

Unit cell/computational domain.

Need to vary $\{\mathbf{a}_1, \mathbf{a}_2\}$ in addition to minimising over ψ



Ground states and excitations

Planar kinetic energy

$$T_v = \frac{(-i\hbar\nabla_{\perp} + m\mathbf{v})^2}{2m}$$

Axial single particle energy

$$H_z = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial z^2} + \frac{1}{2} m \omega_z^2 z^2$$

2-body interactions

$$\Phi(\mathbf{x}) = \int d\mathbf{x}' U(\mathbf{x} - \mathbf{x}') |\psi(\mathbf{x}')|^2$$

Time-independent Gross-Pitaevskii Equation (GPE)

$$\mathcal{L}\psi = \mu\psi$$

(Incl Optimise unit cell)

$$\mathcal{L} = T_0 + H_z + \Phi + \gamma_{QF}\psi^3$$

Superfluid tensor

$$f_{s,ij} = \frac{1}{m\rho} \frac{\partial^2 \mathcal{E}}{\partial v_i \partial v_j}$$

Linearize the time-dependent GPE to determine normal modes:

$$\Psi(\mathbf{x}, t) = e^{-i\mu t/\hbar} \left[\psi(\mathbf{x}) + \sum_{\nu\mathbf{q}} \left\{ c_{\nu\mathbf{q}} u_{\nu\mathbf{q}}(\mathbf{x}) e^{-i\omega_{\nu\mathbf{q}} t} - c_{\nu\mathbf{q}}^* v_{\nu\mathbf{q}}^*(\mathbf{x}) e^{i\omega_{\nu\mathbf{q}}^* t} \right\} \right]$$

Descrete translational symm.: excitations have well-defined quasi-momentum

Bogoliubov-de Gennes Equation (BdG)

$$\begin{pmatrix} \mathcal{L} + X - \mu & -X \\ X & -(\mathcal{L} + X - \mu) \end{pmatrix} \begin{pmatrix} u_{\nu\mathbf{q}} \\ v_{\nu\mathbf{q}} \end{pmatrix} = \hbar\omega_{\nu\mathbf{q}} \begin{pmatrix} u_{\nu\mathbf{q}} \\ v_{\nu\mathbf{q}} \end{pmatrix}$$

$$Xf = \psi \int d\mathbf{x}' U(\mathbf{x} - \mathbf{x}') f(\mathbf{x}') \psi(\mathbf{x}') + \frac{3}{2} \gamma_{QF} \psi^3 f$$

Hydrodynamics

- Description of long-wavelength properties of a system in terms *slow* fields.
- These arise from:

slow/hydrodynamic fields

Number conservation

density: $\rho + \delta\rho$

Conservation laws

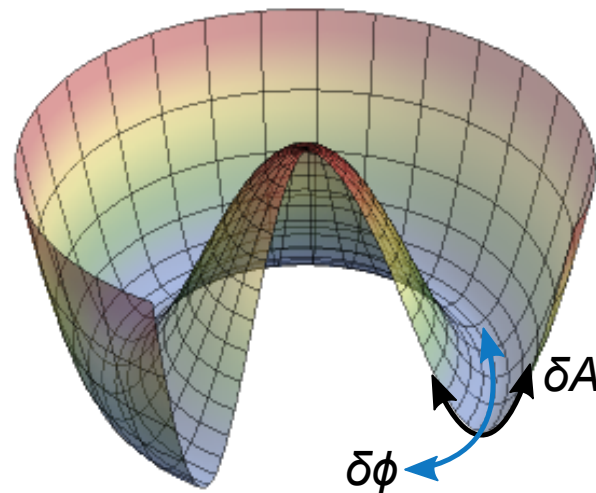
Broken symmetries

U(1) symm.

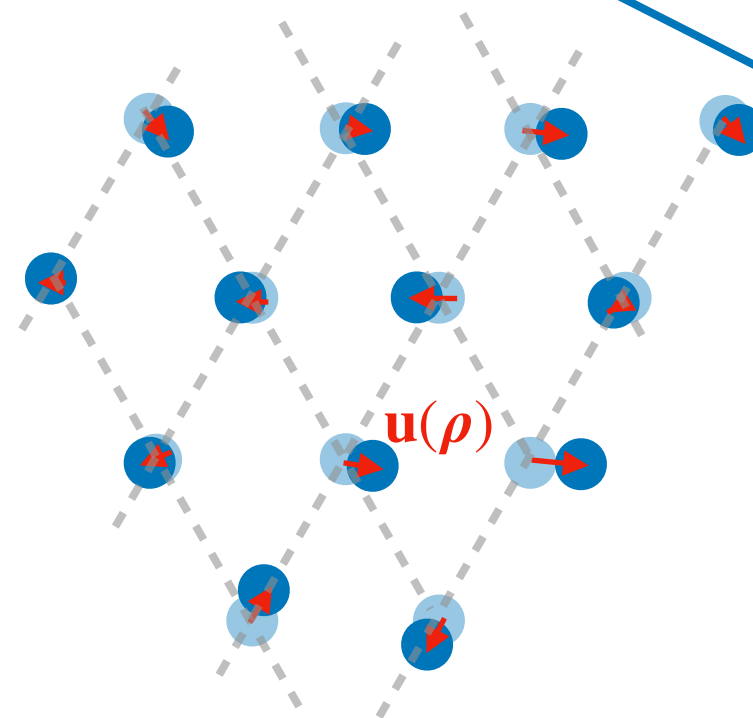
phase: ϕ

superfluid velocity:

$$\mathbf{v}_s = \frac{\hbar}{m} \nabla \phi$$



System	Symmetry	Nambu-Goldstone mode
Fluid	Galilean	Phonon
Solid	Translation	Phonon(s)
BEC/ Superfluid	U(1) (Phase)	Phonon



2D translational symm.

displacement: $\mathbf{u}$

normal velocity:

$$\mathbf{v}_n = \dot{\mathbf{u}}$$

Relevant Elastic properties

How does a supersolid respond to changes in these fields?

density: $\rho + \delta\rho$

$$\alpha_{\rho\rho} = \frac{\partial^2 E}{\partial \rho^2}$$

Bulk modulus

(Fluid compressibility)

Mixed **density-strain**
term (small)

$$\alpha_{\rho u} = \frac{\partial^2 E}{\partial \rho \partial \epsilon_{xx}} \rightarrow \gamma$$

displacement: $\mathbf{u}$

$$C_{ijkl} = \frac{\partial^2 E}{\partial \epsilon_{ij} \partial \epsilon_{kl}}$$

Sym. strain tensor:

$$\epsilon_{ij} \equiv \frac{1}{2}(\partial_i u_j + \partial_j u_i)$$

Elastic tensor (isotropic)

(Solid compressibility)

$$C_{ijkl} = \lambda \delta_{ij} \delta_{kl} + \mu (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk})$$

λ First Lamé parameter

μ Shear modulus

phase: ϕ

$$f_{ij} = \frac{1}{m\rho} \frac{\partial^2 E}{\partial v_{s,i} \partial v_{s,j}}$$

Superfluid tensor

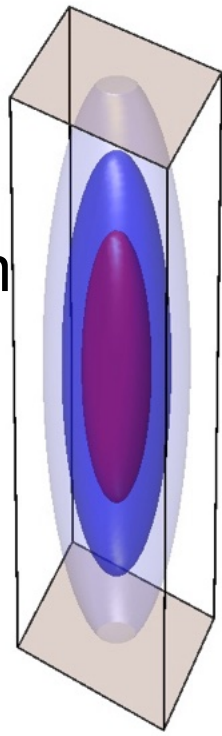
(isotropic)

----- $f_{ij} = f_s \delta_{ij}$

Restrict attention to **isotropic** triangular/honeycomb phases

Ground state energy density (per area) from single cell

$$E(\rho, \mathbf{a}_1, \mathbf{a}_2, \mathbf{v}_s)$$



Calculating elastic parameters

...from ground state calculations

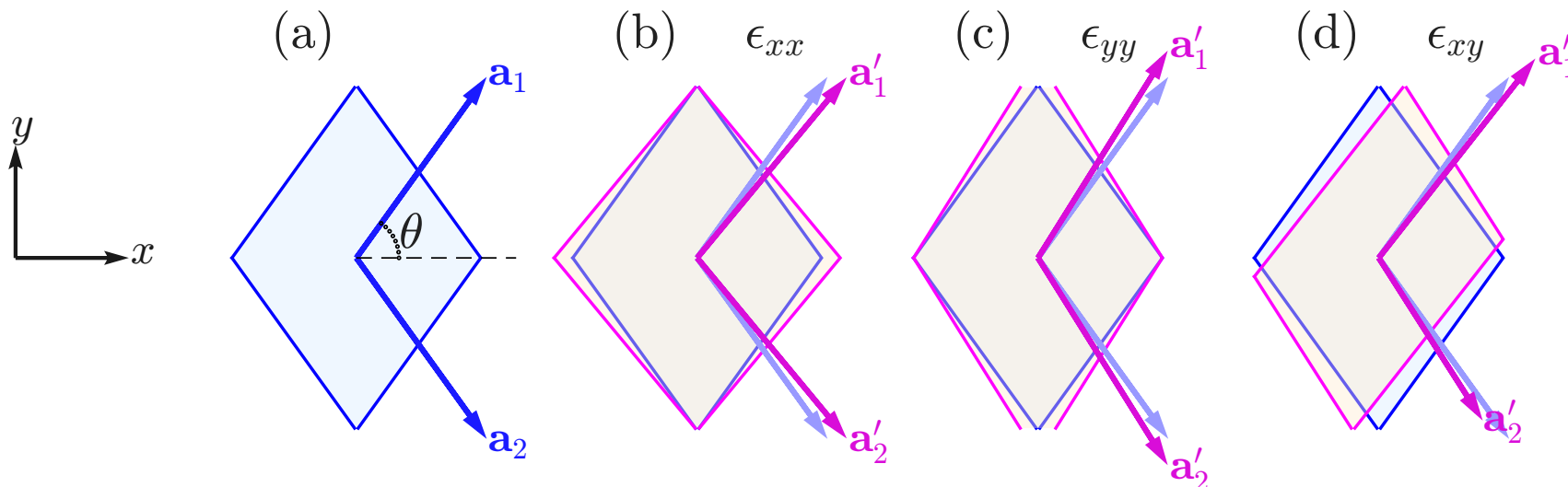
1. Change the density/number of atoms

2. Vary the boundary conditions/velocity

3. Strain the unit cell

$$\alpha_{\rho\rho} = \frac{\partial^2 E}{\partial \rho^2}$$

$$f_{ij} = \frac{1}{m\rho} \frac{\partial^2 E}{\partial v_{s,i} \partial v_{s,j}}$$



$$C_{ijkl} = \frac{\partial^2 E}{\partial \epsilon_{ij} \partial \epsilon_{kl}}$$

4. Mixed differences

$$\alpha_{\rho u} = \frac{\partial^2 E}{\partial \rho \partial \epsilon_{xx}} \rightarrow \gamma$$

Supersolid Lagrangian

Starting from the **Supersolid Lagrangian**:

$$\begin{aligned}\mathcal{L}(\delta\rho, \phi, \mathbf{u}) = & -\hbar\delta\rho\partial_t\phi - \rho\frac{\hbar^2}{2m}(\partial_i\phi)^2 - \gamma\delta\rho\epsilon_{ii} - \frac{1}{2}\alpha_{\rho\rho}(\delta\rho)^2 \\ & + \frac{1}{2}m\rho_n\left(\partial_t u_i - \frac{\hbar}{m}\partial_i\phi\right)\left(\partial_t u_j - \frac{\hbar}{m}\partial_j\phi\right) - \frac{1}{2}C_{ijkl}\epsilon_{ij}\epsilon_{kl}\end{aligned}$$

DT Son PRL 2005; Yoo & Dorsey PRB 2010

One E-L equation trivial: $\delta\rho = -(\hbar\partial_t\phi + \gamma\epsilon_{ii})/\alpha_{\rho\rho}$ can eliminate $\delta\rho$.

Reduced Lagrangian

$$\mathcal{L}(\phi, \mathbf{u}) = \frac{\hbar^2}{2\alpha_{\rho\rho}}(\partial_t\phi)^2 - \rho_s\frac{\hbar^2}{2m}(\partial_i\phi)^2 + \frac{1}{2}m\rho_n(\partial_t u_i)^2 - \hbar\tilde{\rho}\partial_t u_i\partial_j\phi - \frac{1}{2}\tilde{C}_{ijkl}\epsilon_{ij}\epsilon_{kl}$$

Renormalized elastic parameters:

$$\tilde{C}_{ijkl} \equiv C_{ijkl} - \frac{\gamma^2}{\alpha_{\rho\rho}}\delta_{ij}\delta_{kl}, \quad \text{i.e.} \rightarrow \tilde{\lambda} = \lambda - \gamma^2/\alpha_{\rho\rho}$$

$$\tilde{\rho} \equiv \rho_n - \frac{\gamma}{\alpha_{\rho\rho}},$$

Equations of Motion and sound

Reduced Lagrangian

$$\mathcal{L}(\phi, \mathbf{u}) = \frac{\hbar^2}{2\alpha_{\rho\rho}} (\partial_t \phi)^2 - \rho_s \frac{\hbar^2}{2m} (\partial_i \phi)^2 + \frac{1}{2} m \rho_n (\partial_t u_i)^2 - \hbar \tilde{\rho} \partial_t u_i \partial_j \phi - \frac{1}{2} \tilde{C}_{ijkl} \epsilon_{ij} \epsilon_{kl}$$



Euler-Lagrange Eqs

Superfluid hydrodynamics

$$\frac{\hbar^2}{\alpha_{\rho\rho}} \partial_t^2 \phi - \frac{\hbar^2 \rho_s}{m} \nabla^2 \phi = \hbar \tilde{\rho} \partial_t (\nabla \cdot \mathbf{u}),$$

$$m \rho_n \partial_t^2 \mathbf{u} - \mu \nabla^2 \mathbf{u} - (\tilde{\lambda} + \mu) \nabla \nabla \cdot \mathbf{u} = \hbar \tilde{\rho} \partial_t (\nabla \phi)$$

Navier-Cauchy eq. (fundamental eq. of linear elasticity)

"Uncoupled" sounds

Speed of sound fluid

$$m c_s^2 \equiv \rho_s \alpha_{\rho\rho}$$

Normal component coupling between superfluid and crystal

$$c_{\tilde{\rho}}^2 = \tilde{\rho}^2 \alpha_{\rho\rho} / m \rho_n$$

Speeds of sound solid

$$m c_L^2 \equiv \frac{\tilde{\lambda} + 2\mu}{\rho_n}, \quad m c_t^2 \equiv \frac{\tilde{\mu}}{\rho_n}$$

Two Supersolid Longitudinal sounds: first & second sound

$$c_{\pm} = \frac{1}{2} \left[c_s^2 + c_L^2 + c_{\tilde{\rho}}^2 \pm \sqrt{(c_s^2 + c_L^2 + c_{\tilde{\rho}}^2)^2 - 4c_s^2 c_L^2} \right].$$

+ decoupled transverse sound

Speeds of sound

Calculation of elastic properties

Dipolar supersolid highly incompressible $\alpha_{\rho\rho} \gg \alpha_{uu}$

Ground state response

Superfluid fraction: f_s

bulk modulus: $\alpha_{\rho\rho}$

longitudinal modulus:

$$\alpha_{uu} = \tilde{\lambda} + 2\tilde{\mu}$$

shear modulus: $\tilde{\mu}$

elastic params (kHz)

f_s, C

c_+ first sound Honeycomb

c_- second sound

c_t transverse sound

$\alpha_{uu}/\rho h$

$\tilde{\mu}/\rho h$

$\alpha_{\rho u}/h$

Unif.

a_s/a_{dd}

0.765 0.77 0.775 0.78 0.785

10⁻⁴ 10⁻³ 10⁻² 10⁻¹ 10⁰ 10¹

0.6 0.8 1

(a)

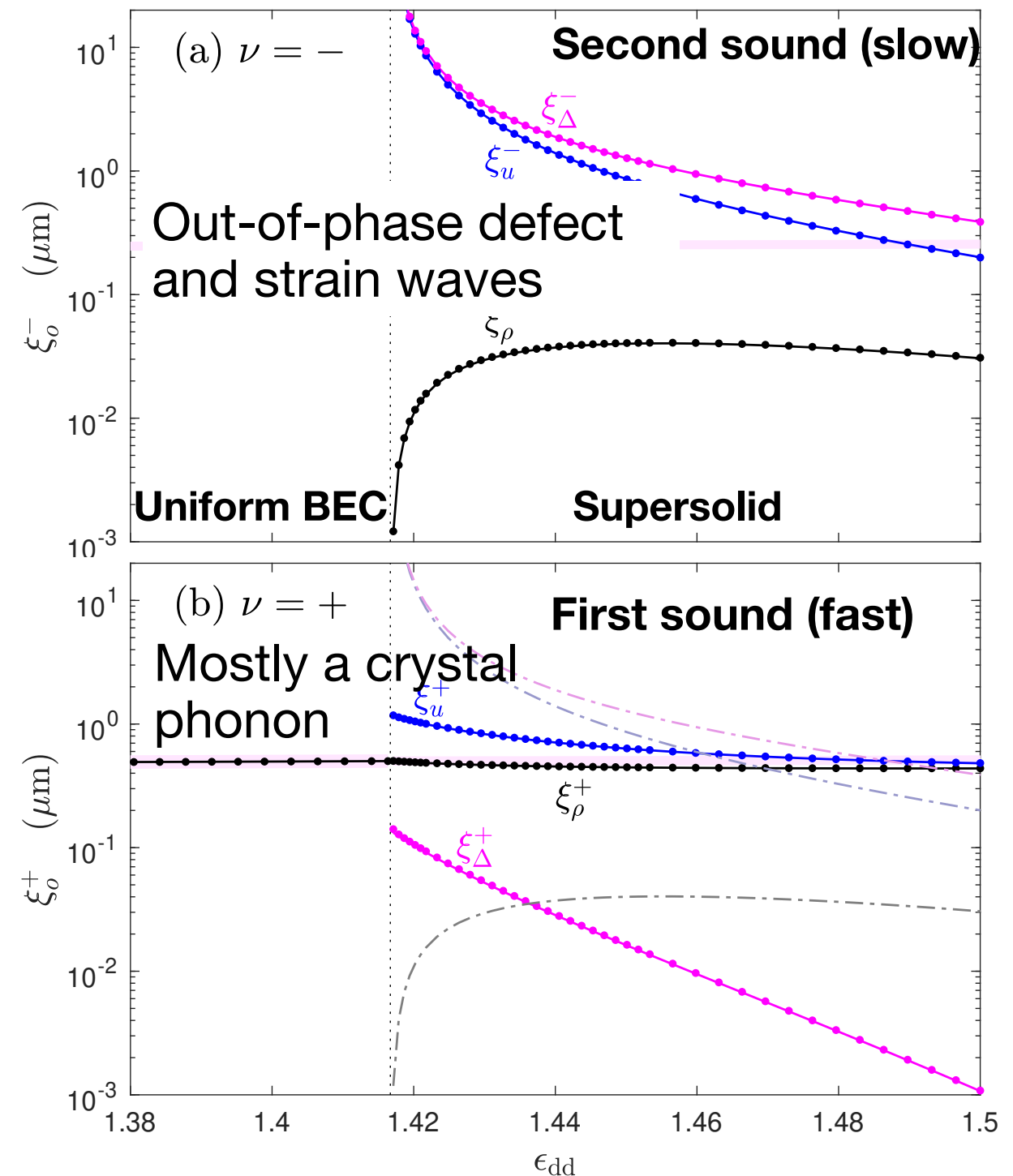
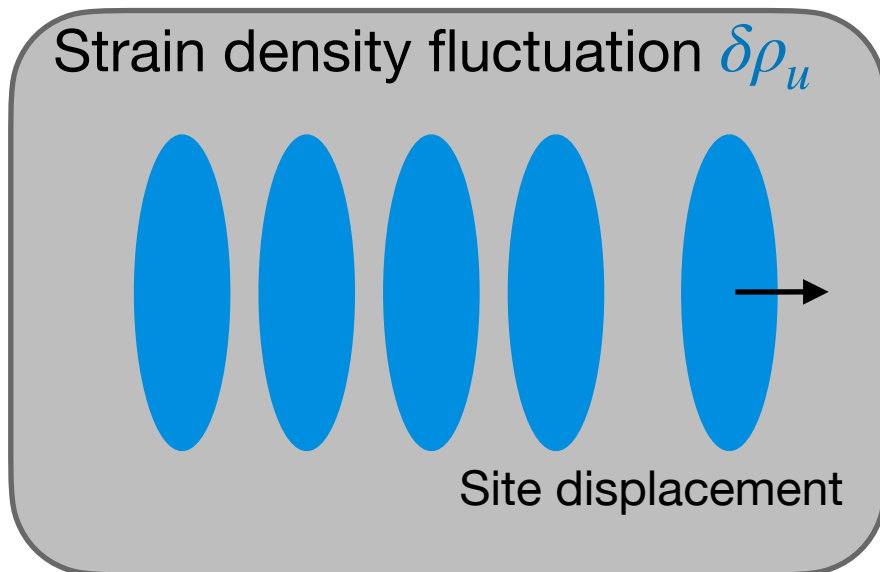
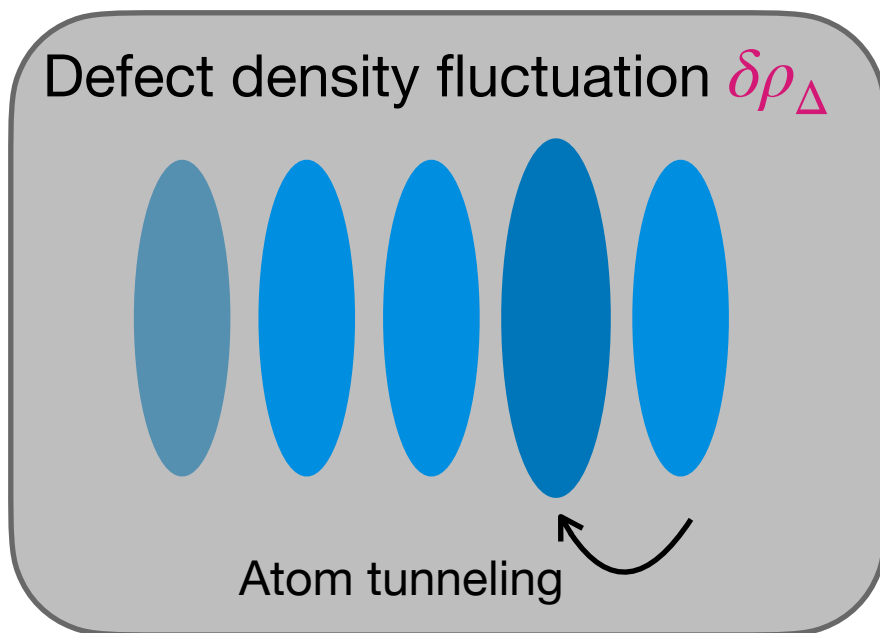
(c)

Character of longitudinal sound

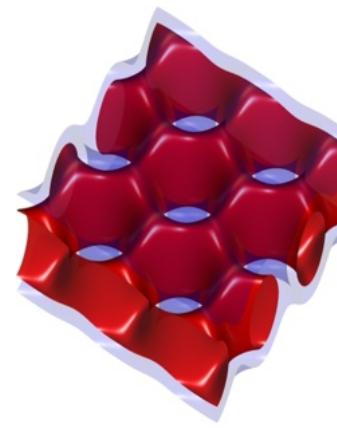
Longitudinal waves contribute to density fluctuations

These results are also analytically determined by hydrodynamics

Decompose density fluctuation $\delta\rho = \delta\rho_u + \delta\rho_\Delta$



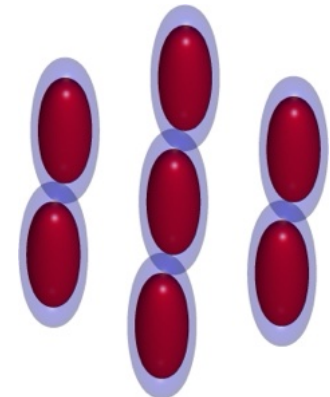
Summary



Honeycomb



Stripe



Triangular

- Intro to dipolar supersolids
- Rich ground state phase diagram
- Band structure $\rightarrow$ sound
- Hydrodynamic theory: slow fields, elastic parameters
- Overview of dynamics and future directions

Relevant publications from our group:

2D Phase diagram: Ripley *et al.*, PRA (2023)

Stripe phase in 2D binary supersolid, Chen *et al.*, PRA (2024)

Sound & fluctuations 1D supersolid: Platt *et al.*, PRA (2024)

Sound in 2D supersolids: Poli *et al.*, PRA (2024)

Sound in binary 1D supersolid: Kirkby *et al.*, PRL (2024)

Honeycomb supersolid: Blakie, PRL (2025)

Anisotropic 2D supersolids: Cook PRA (2026)

